Max-Planck-Institut
für Plasmaphysik

IPP

Scaling Theory for Cross-Field Transport of Cosmic Rays in Turbulent Fields

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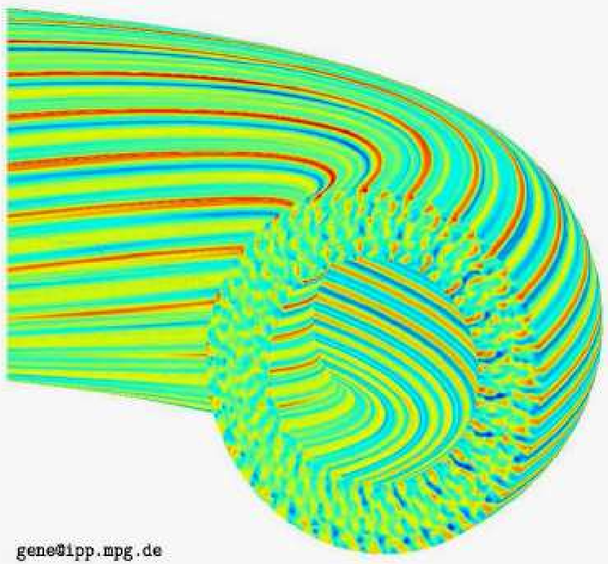
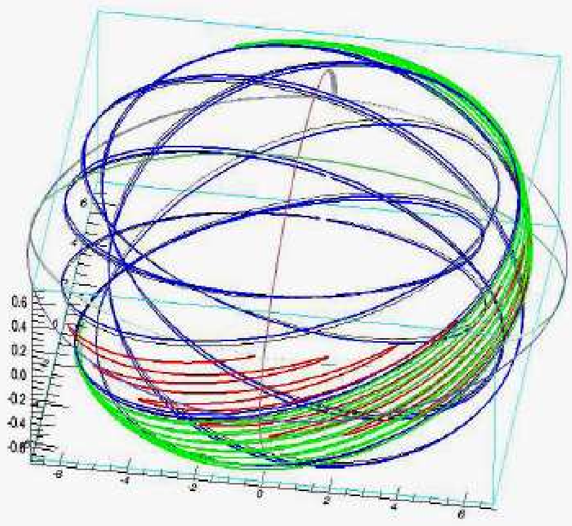
Energetic particle transport in fusion plasmas



Turbulence in a torus (GENE)



Drift orbits in the torus

Transport of (nonrelativistic) energetic ions:
T. Hauff et al., Physical Review Letters **102**, 075004 (2009)

Transport of (relativistic) runaway electrons:
T. Hauff and F. Jenko, Physics of Plasmas **16**, 102308 (2009)

Key idea of this talk

Within the framework of the test particle approach

one can understand turbulent transport scalings

by inspection of decorrelation mechanisms

Particle fieldline diffusion

Basic assumptions

The gyroradii of the particles are small compared to the correlation lengths of the turbulence

Particles have small to moderate pitch angles; they move ballistically along the fieldlines

Connecting the running diffusion coefficient and the Lagrangian velocity autocorrelation function

$$D_x(t) \equiv \frac{1}{2}\frac{d}{dt}\langle x(t)^2\rangle$$
$$= \int_0^t d\xi \, \langle \, v_x(0) v_x(\xi) \, \rangle \equiv \int_0^t d\xi \, L_{v_x}(\xi)$$

Taylor (1920), Green (1951), Kubo (1957)

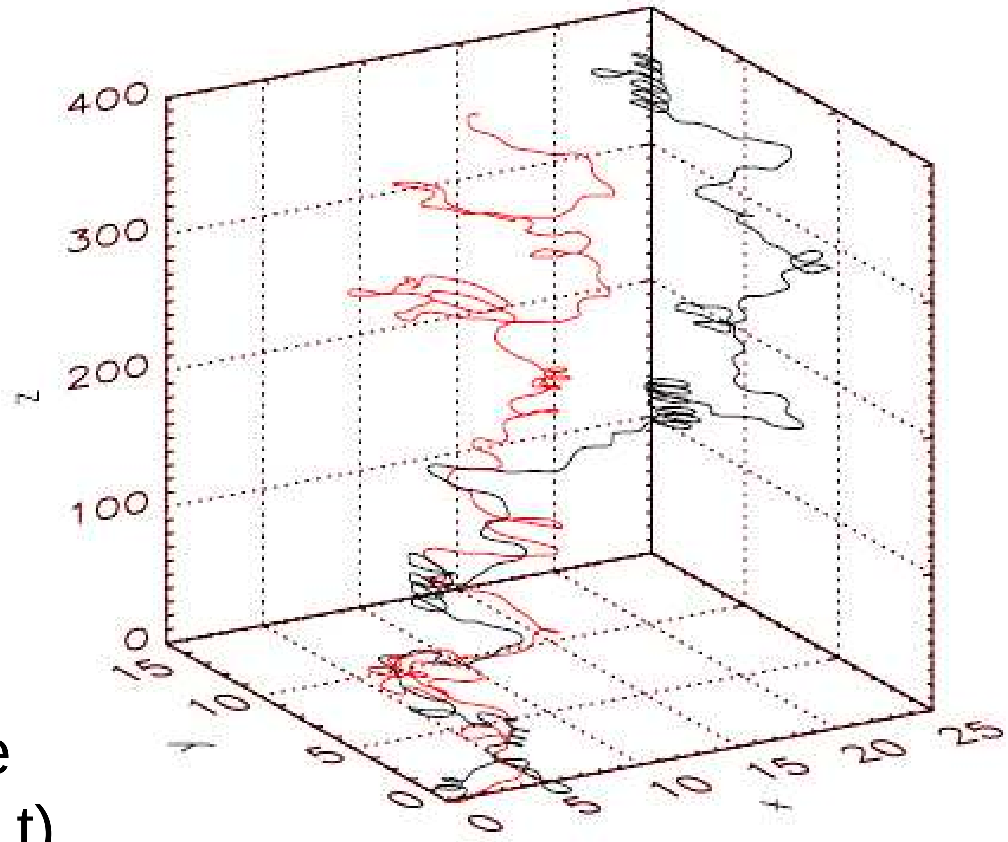
Particles following perturbed fieldlines

$$\tilde{\mathbf{v}}_{0,\perp} = v_\parallel \tilde{\mathbf{B}}_\perp / B_0$$

$$\tilde{B} \lesssim B_0$$

$$\tilde{\mathbf{v}}_{0,\perp} = v_\parallel \frac{\nabla \tilde{A}_\parallel \times \mathbf{e}_z}{B_0}$$

Hamiltonian structure
(dependence on z, not t)



3D case

Kubo number and decorrelation

Kubo number: A ratio of two time scales

Question: Do the particles feel the structures?

$$K = \frac{\tau_c}{\tau_{\mathrm{fl}}} \qquad\qquad K \equiv \frac{V_B \tau_c}{\lambda_\perp} = \frac{\sqrt{\langle \tilde{B}_\perp^2 \rangle}}{B_0} v_\parallel \frac{\tau_c}{\lambda_\perp}$$

In the case of parallel decorrelation:

$$\tau_c = \lambda_\parallel / v_\parallel$$

$$K = \frac{\tilde{B}_\perp}{B_0} \frac{\lambda_\parallel}{\lambda_\perp}$$

Small Kubo numbers

Lagrangian velocity autocorrelation function:

$$L_{v_x}(t) \propto E(z(t) = v_\parallel t) \qquad E(z) = \langle \tilde{A}_\parallel^2 \rangle e^{-z/\lambda_\parallel}$$

$$L_{v_x}(t) = V_B^2 e^{-t/\tau_c} \text{ with } \tau_c = \lambda_\parallel / v_\parallel$$

Resulting particle diffusivities:

$$D_\perp(t) = V_B^2 \tau_c (1 - e^{-t/\tau_c})$$

$$\rightarrow V_B^2 t = (\tilde{B}/B_0)^2 v_\parallel^2 t \quad (t/\tau_c \ll 1)$$

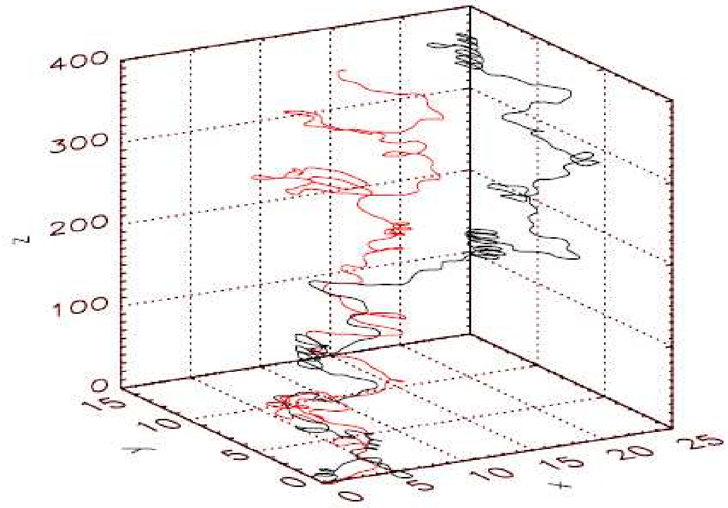
$$\rightarrow V_B^2 \tau_c = (\tilde{B}/B_0)^2 \lambda_\parallel v_\parallel \quad (t/\tau_c \gg 1)$$

Large Kubo numbers

New: Particle trapping events!

$$D_\perp \propto K^\gamma \lambda_\perp^2 / \tau_c \qquad D_\perp \approx V_B^\gamma \lambda_\perp^{2-\gamma} \tau_c^{\gamma-1}$$

$$D_\perp \approx \left(\frac{\tilde{B}}{B_0} \right)^\gamma \lambda_\perp^{2-\gamma} \lambda_\parallel^{\gamma-1} v_\parallel$$



Naïve expectation:

$\gamma = 0$ (strong trapping)

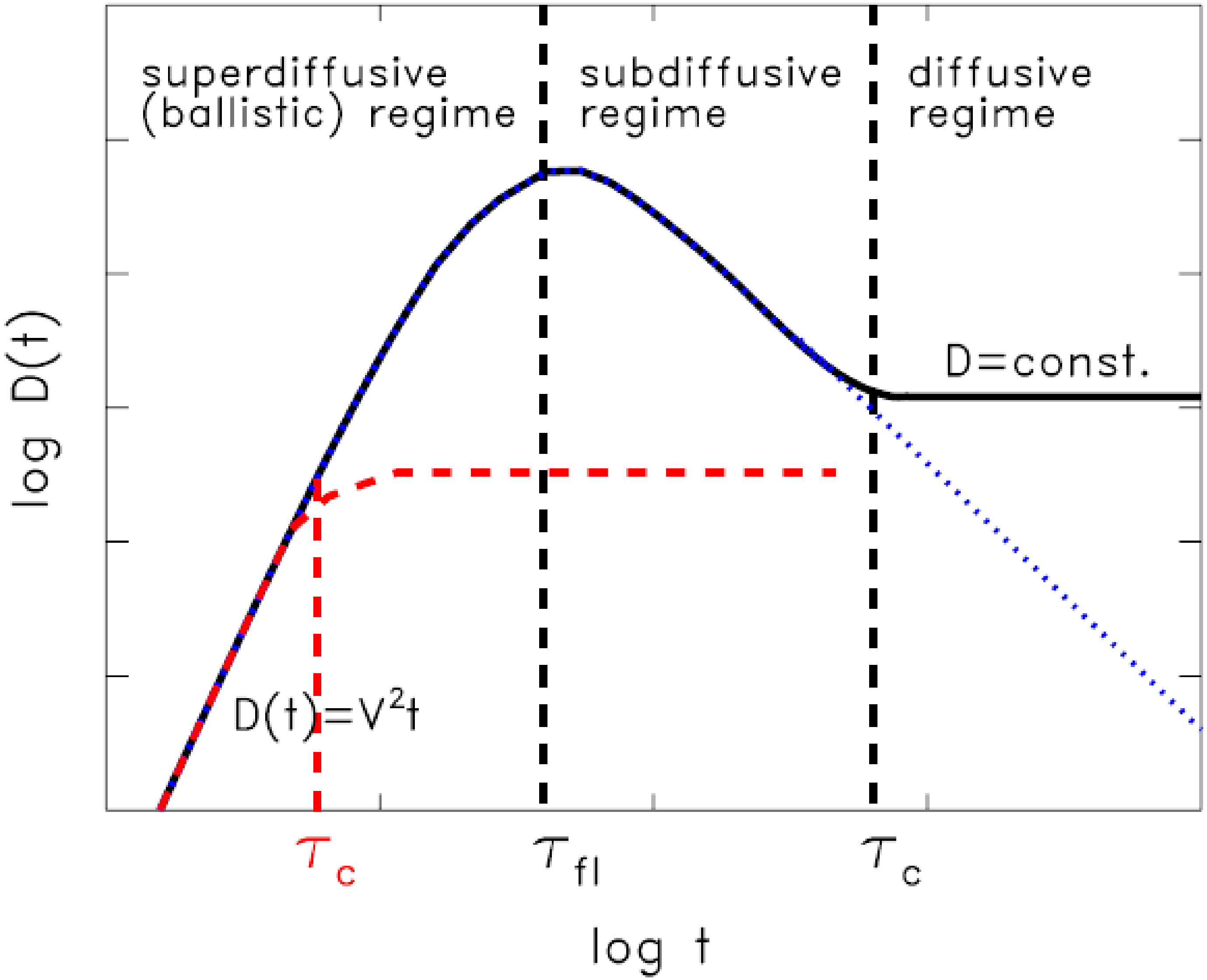
Percolation theory (Gruzinov et al. 1990):

$\gamma = 0.7$ (coexistence of trapped and untrapped particles)

Low Kubo number regime:

$\gamma = 2$ (no trapping)

Running diffusion coefficient and decorrelation



Finite Larmor radius effects

What if the Larmor radii exceed the perpendicular correlation lengths?

There are two possibilities…

Gyro-averaging

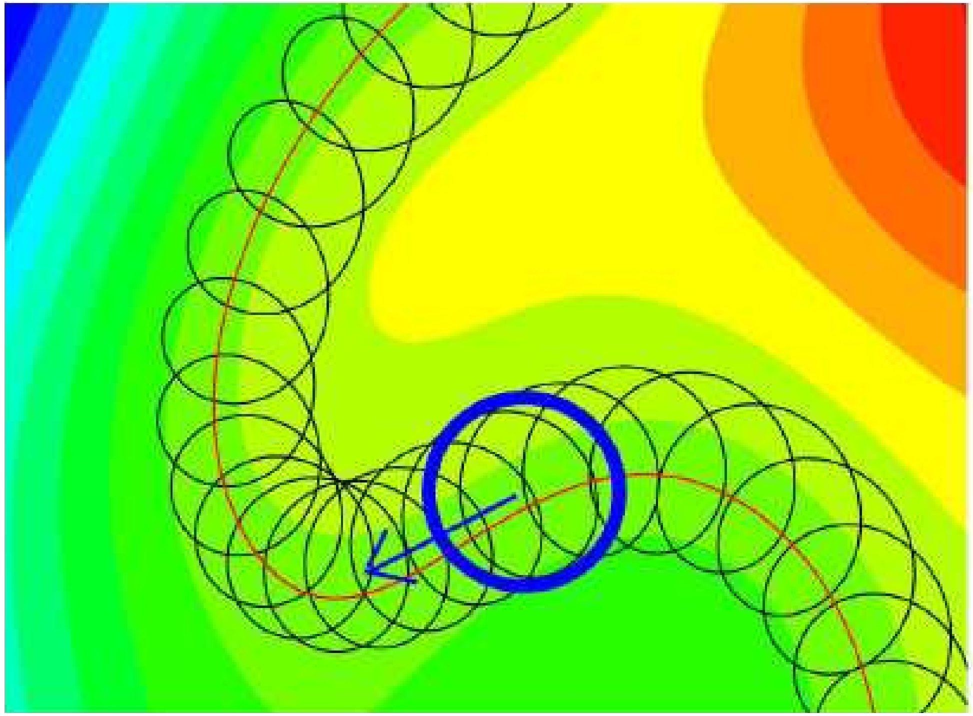
Particles feel a gyroorbit-averaged magnetic field; the drift velocities are reduced accordingly

$$\langle \tilde{A}_\parallel \rangle(\mathbf{x}_0) = \frac{1}{2\pi} \oint d\varphi \, \tilde{A}_\parallel(\mathbf{x}_0 + \rho_\mathbf{g}(\varphi))$$

$$\langle \tilde{A}_\parallel \rangle(\mathbf{x}_0) = \frac{1}{2\pi} \oint d\varphi \sum_\mathbf{k} \tilde{A}_{\parallel,\mathbf{k}} e^{i\mathbf{k}\cdot(\mathbf{x}_0+\rho_\mathrm{g}(\varphi))}$$

$$= \sum_\mathbf{k} \tilde{A}_{\parallel,\mathbf{k}} e^{i\mathbf{k}\cdot\mathbf{x}_0} \frac{1}{2\pi} \oint d\varphi e^{i\mathbf{k}\cdot\rho_\mathrm{g}(\varphi)}$$

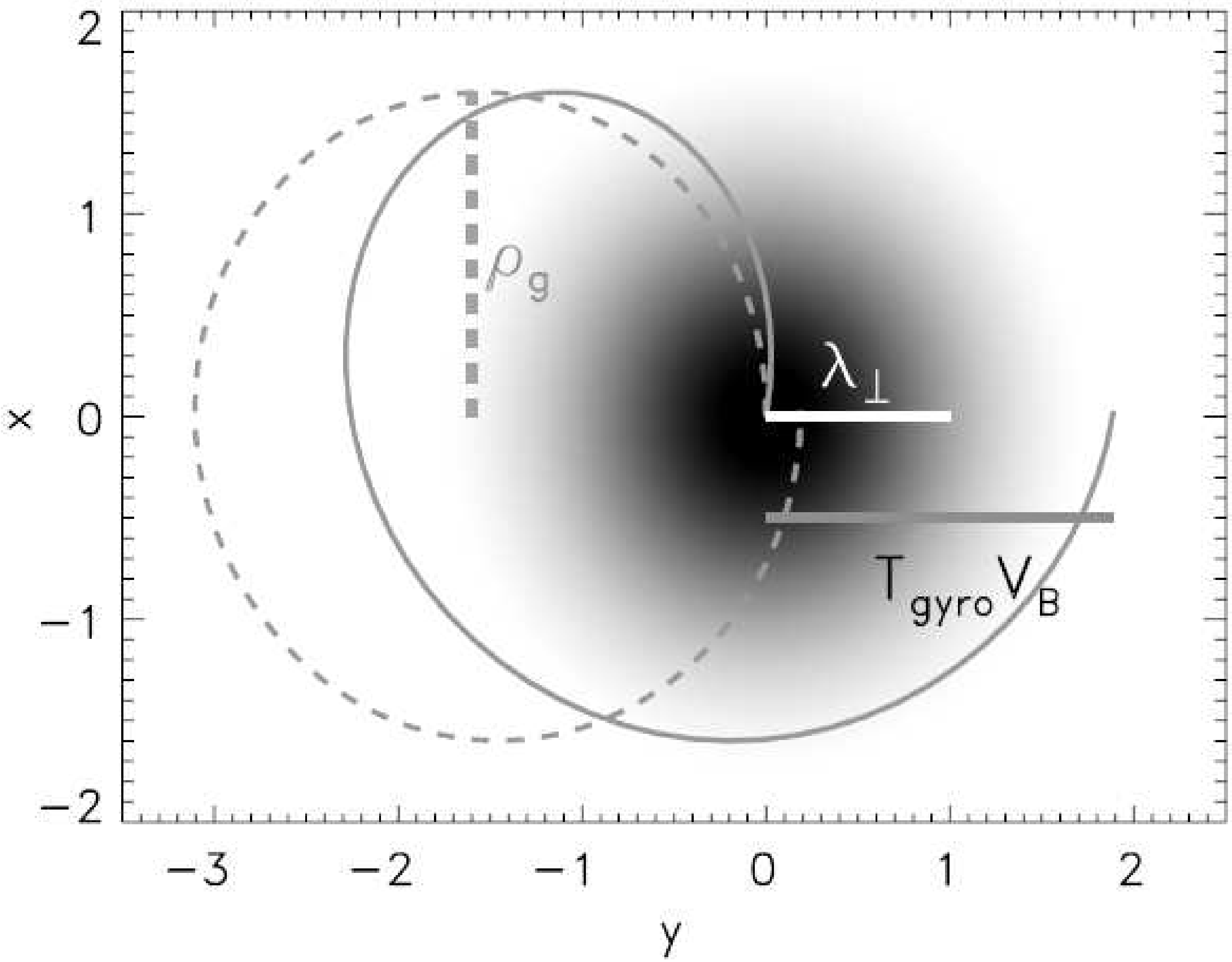
$$= \sum_\mathbf{k} \tilde{A}_{\parallel,\mathbf{k}} \, e^{i\mathbf{k}\cdot\mathbf{x}_0} J_0(k\rho_g) \, .$$



$$V_B^{\mathrm{eff}} = V_B (4\sqrt{\pi}\,\rho_g/\lambda_\perp)^{-1/2}$$

$$T_{\mathrm{gyro}} V_B \ll \lambda_\perp$$

Parallel or perpendicular decorrelation?



Perpendicular decorrelation

Decorrelation time:

$$\tau_{\text{gyro}} = T_{\text{gyro}} \frac{\lambda_\perp}{2\pi \rho_g} = \frac{\lambda_\perp}{v_\perp}$$

Resulting particle diffusivity:

$$D_\perp \approx V_B^\gamma \lambda_\perp^{2-\gamma} \tau_{\text{gyro}}^{\gamma-1} = \left(\frac{\tilde{B}}{B_0} \right)^\gamma v_\parallel^\gamma \lambda_\perp v_\perp^{1-\gamma}$$

Modified Kubo number:

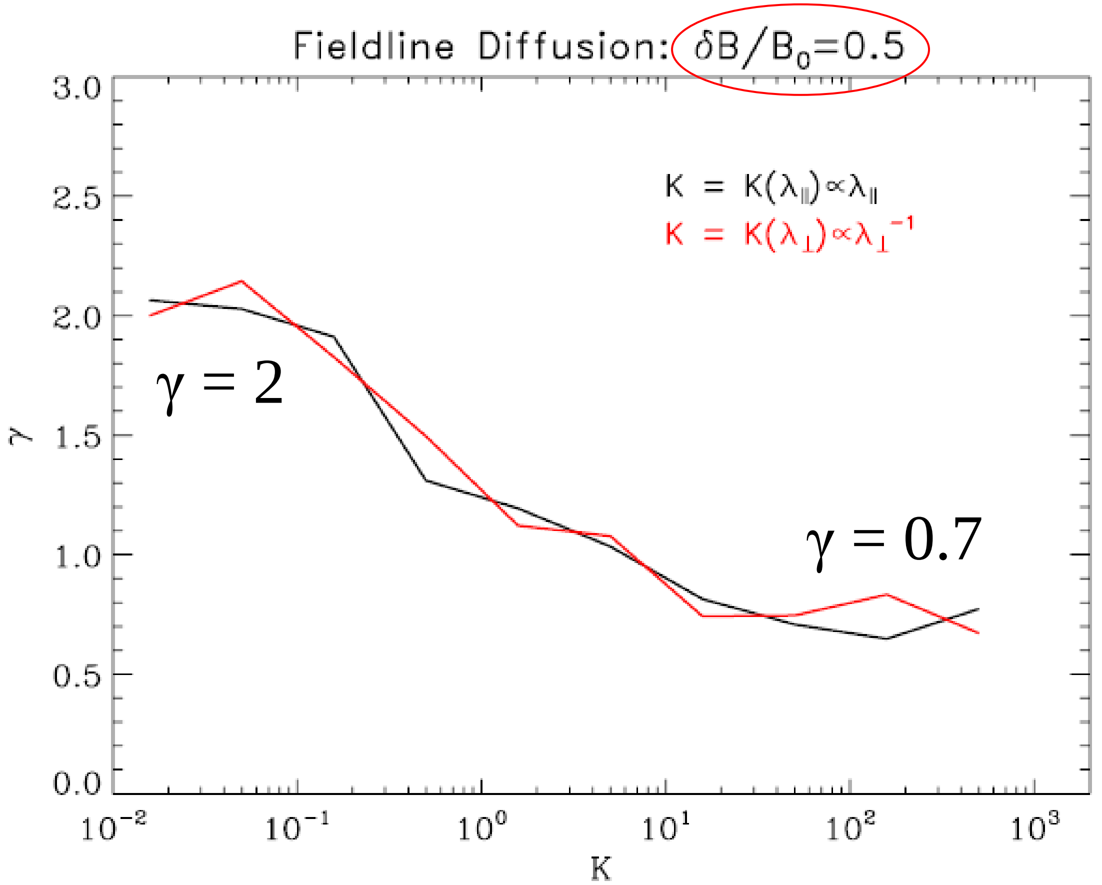
$$K_{\text{gyro}} = \frac{\tilde{B}_\perp}{B_0} \frac{v_\parallel}{v_\perp}$$

Necessary conditions:

$$\rho_g \gtrsim \lambda_\perp \qquad \tau_{\text{gyro}} \lesssim \tau_\parallel = \lambda_\parallel / v_\parallel$$

Direct numerical simulations

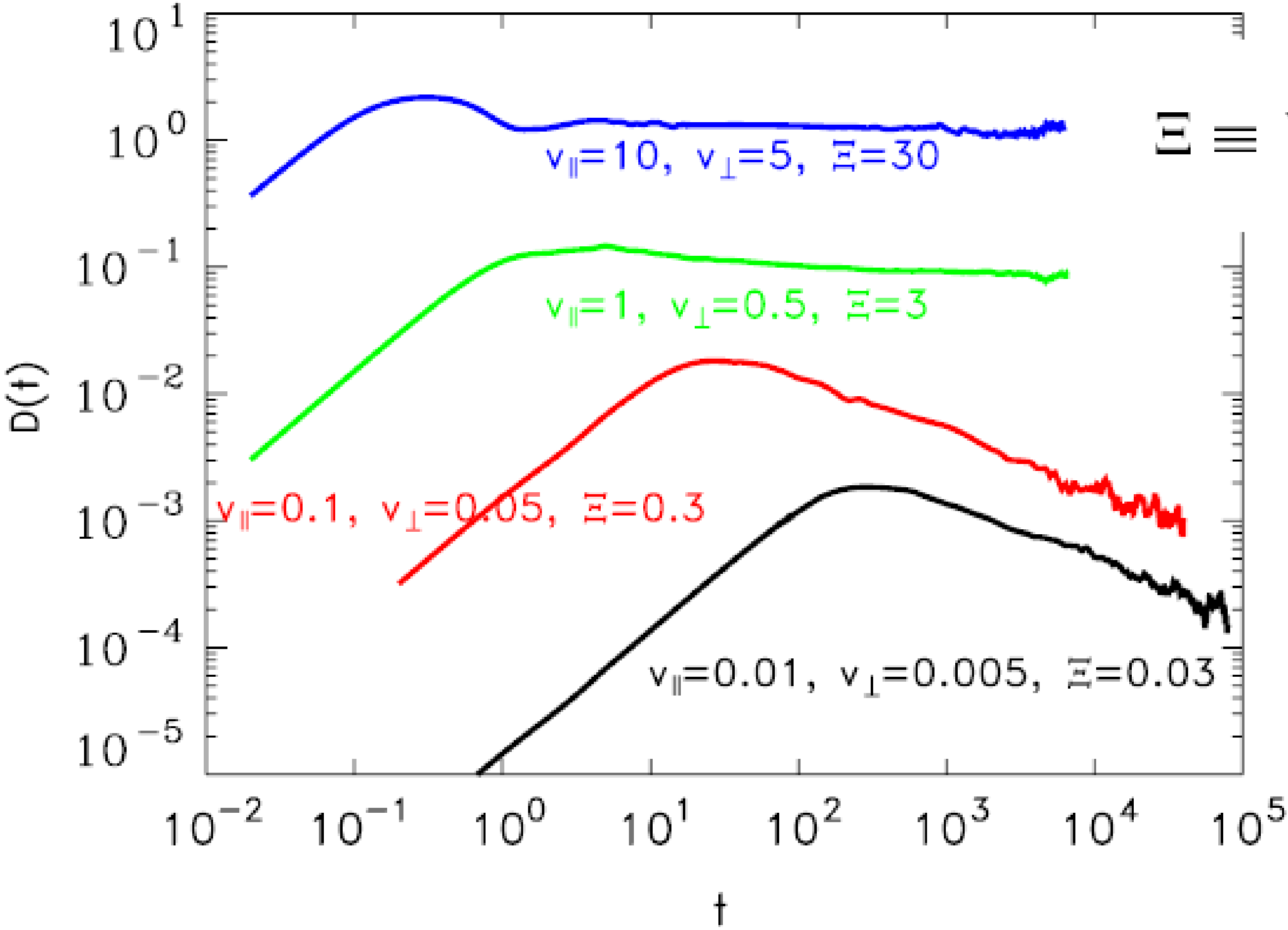
Pseudo-turbulent fields: superposition of around 1000 planar waves
Particles subject to Lorentz force, integration via 4th order ERK method
Pitch angle scattering is fully retained but subdominant for $v_\parallel \gtrsim v_\perp$



See also Zimbardo et al. 2000

Direct numerical simulations (cont'd)

2D turbulence: Running diffusion coefficient in different regimes
Subdiffusion turns into diffusion as particles deviate from fieldlines



$$\Xi \equiv V_B \frac{T_{\rm gyro}}{\lambda_\perp}$$

The relativistic case

Particle fieldline diffusion:

$$D_\perp = \left(\frac{\tilde{B}_\perp}{B_0}\right)^\gamma \mu\, c\, \lambda_\parallel^{\gamma-1} \lambda_\perp^{2-\gamma} \sqrt{1 - 1/\kappa^2}$$

With gyroaverging:

$$D_\perp = \left(\frac{\tilde{B}_\perp}{B_0}\right)^\gamma \frac{\mu}{(1-\mu^2)^{\gamma/4}} c^{1-\gamma/2} \left(\frac{eB}{4\sqrt{\pi}\, m_0}\right)^{\gamma/2}$$
$$\times\, 1.73^{2-\gamma} \lambda_\parallel^{\gamma-1} \lambda_\perp^{2-\gamma/2} \frac{(\kappa^2 - 1)^{1/2-\gamma/4}}{\kappa}$$

Perp. decorrelation:

$$D_\perp = \left(\frac{\tilde{B}_\perp}{B_0}\right)^\gamma \mu^\gamma (1-\mu^2)^{1/2-\gamma/2} c\, \lambda_\perp \sqrt{1 - 1/\kappa^2}$$

...with…

$$\mu \equiv v_\parallel / v = \sqrt{E_\parallel / E} \qquad \kappa \equiv \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} = \frac{E}{m_0 c^2} + 1$$

Summary

We have studied the perpendicular transport of cosmic rays in magnetic turbulence, focusing on regimes where pitch angle scattering effects are subdominant.

The respective physics can be understood by means of careful analysis of the underlying decorrelation mechanisms.

If the Larmor radii exceed the perpendicular correlation lengths, the cross-field transport is set by either gyro-orbit reduction or perpendicular decorrelation.

General scaling relations can be derived which are confirmed by direct numerical simulations.

For more details, please see: ApJ 711, 997 (2010) and www.ipp.mpg.de/~fsj