Ultrahighenergy galactic cosmic rays from distributed focused acceleration

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2. Diffusion approximation
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4. Exemplary spectra of accelerated UHE hadrons
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References:

Cosmic Ray Diffusion Approximation with Weak Adiabatic Focusing; Schlickeiser, R. & Shalchi, A., 2008, ApJ 686, 292

First-order distributed Fermi acceleration of relativistic particles in nonuniform magnetic fields with nonvanishing Alfvenic cross helicity turbulence; Schlickeiser, R., 2009, Modern Phys. Lett. A 24, 1461

Cosmic ray transport in non-uniform magnetic fields: Consequences of gradient and curvature drifts; R. Schlickeiser & F. Jenko, 2010, J. Plasma Phys. 76, 317

1. Fokker-Planck transport equation

The starting point for the transport of cosmic rays in magnetic field $\vec{B}_0(z) = B_0(z)\vec{e}_z$ with superposed weak electromagnetic turbulence $(\delta\vec{E}, \delta\vec{B})$ is the Fokker–Planck equation for the gyrotropic particle phase space density $f_0(X, Y, z, p, \mu, t)$ **per unit of magnetic line length**:

$$\frac{\partial f_0}{\partial t} + v\mu\frac{\partial f_0}{\partial z} - S_0(z,p,t) = \sum_{i,j}\frac{\partial}{\partial x_i}D_{x_i x_j}\frac{\partial f_0}{\partial x_j} - \frac{v}{2L}(1-\mu^2)\frac{\partial f_0}{\partial \mu} \qquad (1)$$

where $x_i \in [\mu, p, X, Y]$ and

$$L^{-1}(z) = -\frac{1}{B_0}\frac{dB_0}{dz} \qquad (2)$$

representing the adiabatic focusing of particles for spatial variations of the guide field $B_0(z)$. $L > 0$ for diverging guide field, $L < 0$ for converging guide field. For uniform fields $B_0 =$const. $\rightarrow L = \infty$. We restrict our analysis to isotropic source terms $S(z, p, t)$.

HERE: Consequences of additional adiabatic focusing term.

2. Diffusion approximation

For low-frequency MHD plasma turbulence: $\delta\vec{E} << \delta\vec{B}$ so that

$$D_{\mu\mu} >> D_{\mu p}, D_{\mu X}, D_{pp} \tag{3}$$

Consequently, the gyrotropic distribution function $f_0(X, Y, z, p, \mu, t)$ due to the dominating pitch-angle diffusion adjusts very quickly to a quasi-equilibrium through pitch-angle diffusion which is close to the isotropic equilibrium distribution $F_0(X, Y, z, p, t)$ per unit of magnetic line length:

$$f_0(X, Y, z, p, \mu, t) = F_0(X, Y, z, p, t) + g_0(X, Y, z, p, \mu, t) \tag{4}$$

where

$$F_0(X, Y, z, p, t) = \frac{1}{2}\int_{-1}^{1} d\mu\, f_0(X, Y, z, p, \mu, t), \tag{5}$$

$$\int_{-1}^{1} d\mu\, g_0(X, Y, z, p, \mu, t) = 0 \tag{6}$$

and where anisotropy $|g_0| \ll F_0$. Substituting Eq. (4) into Eq. (1), averaging over μ and using $|g_0| \ll F_0$ yields

2.1. Diffusion-convection transport equation

Diffusion-convection equation for the isotropic part of the cosmic ray phase space distribution per unit of magnetic line length in the weak ($|L| >> \lambda$) adiabatic focusing limit is

$$\frac{\partial F_0}{\partial t} - S_0(z,p,t) =$$

$$\begin{pmatrix} \frac{\partial}{\partial X} \\ \frac{\partial}{\partial Y} \\ \frac{\partial}{\partial z} \end{pmatrix} \cdot \begin{pmatrix} \kappa_{XX} & \kappa_{XY} & -\kappa_{zX} \\ \kappa_{YX} & \kappa_{YY} & -\kappa_{zY} \\ \kappa_{zX} & \kappa_{zY} & \kappa_{zz} \end{pmatrix} \begin{pmatrix} \frac{\partial F_0}{\partial X} \\ \frac{\partial F_0}{\partial Y} \\ \frac{\partial F_0}{\partial z} \end{pmatrix} + \frac{1}{p^2}\frac{\partial}{\partial p}\left(p^2 A_D \frac{\partial F_0}{\partial p}\right)$$

$$+\frac{v}{4}\frac{\partial}{\partial z}\left(a_{11}\frac{\partial F_0}{\partial p}\right) - \frac{1}{4p^2}\frac{\partial}{\partial p}\left(p^2 v a_{12}\frac{\partial F_0}{\partial z}\right) + \sum_{i=1,2}\left(\frac{1}{p^2}\frac{\partial}{\partial p}p^2 a_{21}\frac{\partial F_0}{\partial X_i} + \frac{\partial}{\partial X_i}a_{22}\frac{\partial F_0}{\partial p}\right)$$

$$+\frac{\kappa_{zz}}{L}\frac{\partial F_0}{\partial z} + \sum_{i=1,2}\frac{\kappa_{zi}}{L}\frac{\partial F_0}{\partial X_i} + \frac{v}{4}\frac{a_{11}}{L}\frac{\partial F_0}{\partial p} \tag{7}$$

with the pitch-angle averaged transport parameters

$$\kappa_{zz} = \frac{v\lambda}{3} = \frac{v^2}{8}\int_{-1}^{1} d\mu \frac{(1-\mu^2)^2}{D_{\mu\mu}(\mu)}, \tag{8}$$

$$\kappa_{ij} = \frac{1}{2}\int_{-1}^{1} d\mu [D_{X_i X_j} - \frac{D_{X_i\mu}D_{\mu X_j}}{D_{\mu\mu}(\mu)}], \tag{9}$$

$$\kappa_{zi} = \frac{v}{4}\int_{-1}^{1} d\mu \frac{(1-\mu^2)D_{X_i\mu}}{D_{\mu\mu}(\mu)}, \tag{10}$$

$$A_D = \frac{1}{2}\int_{-1}^{1} d\mu \Big[D_{pp}(\mu) - \frac{D_{\mu p}(\mu)D_{p\mu}(\mu)}{D_{\mu\mu}(\mu)}\Big], \tag{11}$$

$$a_{11} = \int_{-1}^{1} d\mu \frac{(1-\mu^2)D_{\mu p}(\mu)}{D_{\mu\mu}(\mu)}, \tag{12}$$

$$a_{12} = \int_{-1}^{1} d\mu \frac{(1-\mu^2)D_{p\mu}(\mu)}{D_{\mu\mu}(\mu)}, \tag{13}$$

$$a_{21} = \frac{1}{2}\int_{-1}^{1} d\mu \Big[D_{pX_i}(\mu) - \frac{D_{p\mu}(\mu)D_{\mu X_i}}{D_{\mu\mu}(\mu)}\Big], \tag{14}$$

and

$$a_{22} = \frac{1}{2}\int_{-1}^{1} d\mu \Big[D_{X_ip}(\mu) - \frac{D_{\mu p}(\mu)D_{X_i\mu}}{D_{\mu\mu}(\mu)}\Big], \tag{15}$$

respectively.

2.2. Focused acceleration due to weak adiabatic focusing

Adiabatic focusing gives rise to the last three terms in Eq. (7) that represent convective transport terms parallel to the guide field, perpendicular to the guide field and in momentum space, respectively. In the limit $L \to \infty$ of negligible adiabatic focusing these three new terms vanish.

The convective term along the guide field has been derived before by Earl (1976) and Kunstmann (1979); the other two are new. The respective convective speeds depend on the ratio of the corresponding diffusion coefficients or adiabatic deceleration rate to the focusing length.

Particularly interesting is the new convection term in momentum space:

$$\frac{va_{11}}{4L}\frac{\partial F_0}{\partial p} = \frac{V_A H}{3L} p \frac{\partial F_0}{\partial p}$$

For positive values of the product $a_{11}L > 0$ it represents a continuous momentum loss term, whereas for negative values $a_{11}L < 0$ it represents a first-order Fermi-type acceleration term. The focusing length $L(z)$ is positive for a diverging guide magnetic field (see Eq. (2)) and negative for a converging guide field. On the other hand, the absolute value and the sign of the deceleration rate a_{11} depend sensitively on the cross helicity and magnetic helicity of the magnetic field turbulence.

3. Distributed focused acceleration in the galaxy

Radio continuum surveys of our own and external disk galaxies (for review see Sofue et al. 1986) reveal large-scale spatial variations of the galactic guide magnetic field perpendicular to the galactic plane suggesting the exponential variation $B(z) = B_0 \exp(-z/z_b)$ with values of z_b of a few hundred parsecs. . For the exponentially diverging galactic guide magnetic field the focusing length $L = z_b$ is positive and constant, Negative values of the Alfvenic corss helicity H_c will therefore lead to distributed first-order Fermi acceleration of cosmic rays provided that the acceleration rate dominates all continous momentum loss processes of cosmic ray particles.
In axisymmetric isospectral undamped slab Alfvenic turbulence with equal polarisation states of f- and b-Alfven waves the steady-state focused diffusion transport equation for $F_0 = FB(z)$ is

$$\kappa_{zz}\frac{\partial^2 F}{\partial z^2} - \left[\frac{\kappa_{zz}}{L} + V_A H\right]\frac{\partial F}{\partial z} + \frac{1}{p^2}\frac{\partial}{\partial p}\left(p^2 A_D \frac{\partial F}{\partial p} + \frac{HV_A}{3L}p^3 F + p^2 \dot{p}_{\rm loss} F\right) - \frac{F}{T_c} = -S(z,p) \qquad (16)$$

3.1. Rigidity restriction

The transport terms involving the focusing length L result from the conventional guiding center equations of motion (Northrop 1963) which are valid for focusing length $\mathtt{L} \gg r_g$ much larger than the gyroradii r_g of cosmic ray particles which limits the cosmic ray particle rigidity to values

$$\frac{p}{\mathtt{Z}} \ll \frac{eB\mathtt{L}}{c} = 1.1 \cdot 10^{18} (B/4\ \mu\mathrm{G})(\mathtt{L}/300\ \mathrm{pc})\ \ \frac{eV}{c} \tag{17}$$

Note: for Fe-ions ($Z = 28$) rigidity upper limit corresponds to total energy limit

$$E_{\mathrm{tot,Fe}} \ll 3.0 \cdot 10^{19} (B/4\ \mu\mathrm{G})(\mathtt{L}/300\ \mathrm{pc})\ \ \mathrm{eV} \tag{18}$$

Applying the diffusion approximation to cosmic rays at energies above 10^{16} eV is justified by the small measured anisotropies at these energies (Antoni et al. 2004, Hörandel et al. 2006).

3.2. Conditions for distributed first-order Fermi acceleration

For negative cross helicity values $0 > H = -h$, $h > 0$, the term

$$\mathcal{L} = \frac{1}{p^2}\frac{\partial}{\partial p}\left[\frac{HV_A}{3L}p^3F\right] = -\frac{1}{p^2}\frac{\partial}{\partial p}\left[\frac{hV_A}{3L}p^3F\right] \tag{19}$$

in the transport equation (16) describes first-order Fermi acceleration with the acceleration rate

$$\dot{p}_{\rm acc} = a_1 p, \quad a_1 = \frac{hV_A}{3L} = 3\cdot 10^{-14}\frac{b_4 h}{L_{300}n_{-3}^{1/2}} \quad \mathsf{s}^{-1} \tag{20}$$

provided that this rate is larger than the particle's loss rate

$$\dot{p}_{\rm acc} > \dot{p}_{\rm loss} \tag{21}$$

3.2.1. Hadrons

For hadrons $a_1 > b_\pi n_{\rm gas}$ is always larger than the pion production loss rate. The Coulomb and ionisation losses then define a threshold momentum value p_c, given by $a_1 = b_I n_{\rm gas}/p_c^3$ so that

$$p_c = \left(\frac{Z^2 b_I n_{\rm gas}}{a_1}\right)^{1/3} = 0.17 Z^{2/3}\left(\frac{n_{\rm gas}L_{300}}{b_4 h}\right)^{1/3} n_{-3}^{1/6}\ \frac{\mathsf{GeV}}{c} \tag{22}$$

whose value depends only weakly on the assumed interstellar gas parameters. All hadrons with momenta above p_c will undergo this first-order acceleration process up to momenta determined by condition (17) which covers about 10 orders of magnitude.

3.2.2. Electrons

For electrons the acceleration condition (21) can only be fulfilled in the narrow momentum range (ATIC and PAMELA excess!)

$$p_{c1} \leq p \leq p_{c2} \tag{23}$$

with

$$p_{c1} \simeq \frac{\delta_1 n_{\rm gas}}{a_1} = 10^{-2} \frac{n_{\rm gas} L_{300} n_{-3}^{1/2}}{b_4 h} \quad \text{GeV}/c \tag{24}$$

and

$$p_{c2} \simeq \frac{a_1}{\delta_3 n_{\rm gas}} = 300 \frac{b_4 h}{n_{\rm gas} L_{300} n_{-3}^{1/2}} \quad \text{GeV}/c \tag{25}$$

Obviously, **we have identified a distributed first-order Fermi acceleration process that preferentially accelerates relativistic hadrons over ten orders of magnitude in momentum values**, whereas electrons are accelerated over 4 orders of magnitude in momentum values from 0.01 to 300 GeV/c.

4. Exemplary spectra of accelerated UHE hadrons

4.1. Neglect of constant convection speed and momentum diffusion

Introduce the dimensionless momentum variable $x = \frac{p}{m_p c} = \frac{p_t}{A m_p c}$. With nominal interstellar parameter values the focused convection speed ($\alpha = A/Z$)

$$\frac{\kappa_0}{L}\alpha^\eta x^\eta = 5.7 \cdot 10^5 \frac{(\alpha x)^\eta}{(1-\sigma^2)L_{300}} \ \text{cm/s} \tag{26}$$

is larger than the constant convection speed $hV_A = 2.75 \cdot 10^7 h$ cm/s for

$$x > 1.1 \cdot 10^5 [hL_{300}(1-\sigma^2)]^3/\alpha, \tag{27}$$

where we adopt a value of $\eta = 1/3$. For momenta $p > 1.1 \cdot 10^5 A m_p c/\alpha \simeq 10^5/\alpha$ GeV/c, we may neglect the constant convection speed as compared to the focused convection speed.

For a degenerate cross helicity value $h = -H = 1$, which provides the maximum focused acceleration rate (20), momentum diffusion does not occur ($A_D = 0$). The transport equation (16) then reduces to

$$\kappa_0 \frac{\partial}{\partial z}\left[\frac{\partial F}{\partial z} - \frac{F}{L}\right] - \frac{a_1}{\alpha^\eta x^{2+\eta}}\frac{\partial}{\partial x}\left(x^3 F\right) - \frac{F}{\alpha^\eta x^\eta T_c} = -\frac{S_1(z)S_2(p)}{\alpha^\eta x^\eta} \tag{28}$$

4.2. UHE hadrons

We assume that cosmic ray point sources in the Galaxy inject hadrons with the power law distribution $S_2(x) = S_2 x^{-k}$ with the same spectral index k for $x_L \leq x \leq x_M$ up to some maximum momentum $p_{\max} = m_p c x_M$ of the order of 10^{15} eV/nucl.c, corresponding to x_M. The hadron momentum spectrum at higher momenta $x > x_M$ at the position of the solar system ($z = 0$) then is

$$F(z = 0, x > x_M) = \sum_{n=1}^{\infty} W_n H_n(x) \tag{29}$$

with weighting constants W_n and the momentum-dependent function

$$H_n(x) = x^{-(3+\psi)} \exp\left[-\left(\frac{x}{x_c(n)}\right)^{\eta}\right], \tag{30}$$

with

$$\psi(A > 1) = \frac{1}{a_1 T_c} = 0.167 A^{0.7} \frac{L_{300} n_{\mathrm{gas}} n_{-3}^{1/2}}{b_4 h}, \tag{31}$$

and the characteristic dimensionless momentum

$$x_c(n) = \left(\frac{\eta}{\Lambda_n}\right)^{1/\eta} \tag{32}$$

The lowest eigenfunction $H_1(x)$ extends to the highest momenta.

Fokker-Planck . . .
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In terms of the characteristic momentum (for $\eta = 1/3$)

$$x_c = x_c(1) = (3\Lambda_1)^{-3} = \frac{10^4}{\alpha} \tag{33}$$

we obtain

$$H_1(x) = W_1 x^{-s} \exp\left[-\left(\frac{x}{x_c}\right)^{\eta}\right], \tag{34}$$

which is a power-law with a slowly decreasing exponential function.
The power law spectral index (Θ denotes the spep function)

$$s = 3 + \psi = 3 + \frac{A^{0.7}}{6}\Theta[A-1] \tag{35}$$

depends on the cosmic ray species. Because cosmic ray protons undergo no fragmentation or spallation losses $\psi = 0$, their power law index $s_{\rm protons} = 3$ is the smallest. Heavier cosmic ray hadrons have a larger index

$$s(A) = 3 + \frac{A^{0.7}}{6}, \tag{36}$$

because the spallation and fragmentation time scale decreases with mass number. Consequently, the power-law spectral index increases with increasing mass number A. In Table 1 we calculate the phase space density spectral index values for cosmic ray hadrons of different mass number.

Table 1: Phase space density spectral index s for different cosmic ray hadrons

Particle	mass number A	spectral index $s(A)$
Protons	1	3.0
α-particles	4	3.44
C	12	3.95
O	16	4.16
Fe	52	5.65

In Fig. 1 we show the differential number density momentum spectra $N(x) = 4\pi x^2 H_1(x)$, normalized at x_M, for $\eta = 1/3$ for protons and ions:

$$\frac{N(x)}{N(x_M)} = \left(\frac{x}{x_M}\right)^{2-s(A)} \exp\left[-\left(\frac{x-x_M}{x_c}\right)^{1/3}\right] \tag{37}$$

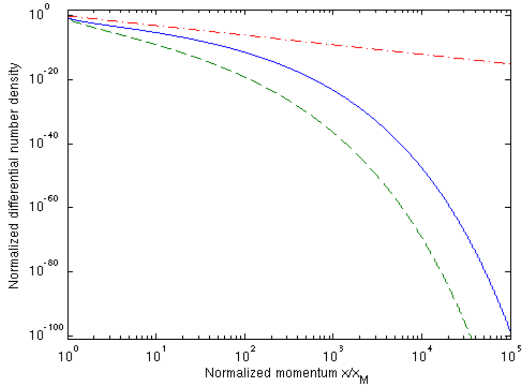

Figure 1: Normalized differential number density momentum spectrum of cosmic ray protons (blue full curve), cosmic ray Fe (dashed green curve) from distributed focused acceleration in comparison with p^{-3}-power law (dot-dashed red curve).

It is remarkable how well the slowly decreasing exponential variation matches a p^{-3}-power law spectrum over a large momentum range.

Fig. 2 shows the deviations of the differential number density momentum spectra from a p^{-3}-power law spectrum for protons, α-particles and iron, illustrating again the good match of the slowly decreasing exponential variation.

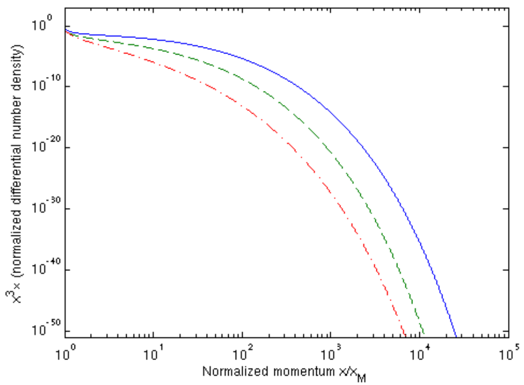

Figure 2: Deviations from a p^{-3}-power law of the normalized differential number density momentum spectrum of cosmic ray protons (blue full curve), cosmic ray Helium (dashed green curve) and cosmic ray Fe (dot-dashed red curve).

5. Summary and conclusions

- The cosmic ray diffusion approximation in the weak adiabatic focusing limit gives rise to three new convective terms (parallel to the guide field, perpendicular to the guide field and in momentum space) in the diffusion-convection transport equation of cosmic rays.

- For positive values of the product $a_{11}L > 0$ the new momentum convection term represents a continuous momentum loss term, whereas for negative values $a_{11}L < 0$ it represents a first-order Fermi-type acceleration term. The focusing length $L(z)$ is positive for a diverging guide magnetic field and negative for a converging guide field. The absolute value and the sign of the deceleration rate a_{11} depend sensitively on the cross helicity H_c and magnetic helicity of the magnetic field turbulence.

- The new 1st-order Fermi acceleration process operates at all cosmic sites where the product $H_cL < 0$ is negative.

- In spiral galaxies cosmic ray hadrons are preferentially accelerated over cosmic ray electrons which have larger continous momentum loss processes. At ultrahigh energies hadron momentum spectra with slowly decreasing exponentials result that closely match a simple p^{-3}-power law spectrum.

6. Physics explanation of the new 1st-order Fermi acceleration

New 1st-order Fermi acceleration is closely related to two effects:
(1) within the adiabatic guiding-center approximation of the transport of charged particles (e. g. Boyd and Sanderson 1969, Rossi and Olbert 1970), the magnetic moment of charged particles is an adiabatic invariant

$$\mu_M = \frac{p_\perp^2}{2m\gamma B(z)} = \frac{pv(1-\mu^2)}{2B(z)} = \text{const.} \tag{38}$$

in a slowly varying guide magnetic field $\mathtt{L} \gg r_g$, with the particles' gyroradius r_g and the focusing length $L^{-1} = -\frac{1}{B(z)}\frac{dB(z)}{dz}$;
(2) if the physical system contains magnetohydrodynamic plasma waves such as Alfven waves, whose magnetic field component is much larger than their electric field component, the quickest particle-wave interaction process is pitch-angle scattering, so that the charged particle distribution function is isotropised on a very short time scale $\tau_{\text{iso}} \ll L/v$. Averaging the magnetic moment (38) with respect to the cosine of pitch-angle μ then yields for the respective quantities at the two positions $z = 0$ and z

$$\frac{< pv >_z}{< pv >_0} = \frac{B(z)}{B(0)} = \exp(-z/L) \tag{39}$$

where in the last step we assume an exponentially varying guide magnetic field.

If the intensities of forward (with parallel phase speed $+V_A$) moving Alfven waves (I^+) and backward (with parallel phase speed $-V_A$) moving Alfven waves (I^-) is different, the resulting net cross helicity of Alfven waves $H = (I^+ - I^-)/(I^+ + I^-)$ results in a net convection speed $V_N = HV_A$ of charged particles as each Alfvenic wave mode isotropises the particles in its rest frame. As a consequence, the average particle position convects as $z = 0 + V_N t = HV_A t$ so that according to Eq. (39) the particle momentum

$$< pv >_z (t) =< pv >_0 \exp\left[-\frac{HLV_A}{L^2}t\right] \tag{40}$$

increases exponentially with time if $HL < 0$ or decreases exponentially with time if $HL < 0$. For relativistic particles ($v \simeq c$) Eq. (40) implies the momentum acceleration rate $\dot{p}/p = -HV_A/L$ which apart from a factor 3 agrees with the exact rate.

This novel distributed 1st order Fermi acceleration process operates in all cosmic sources with $HL < 0$.

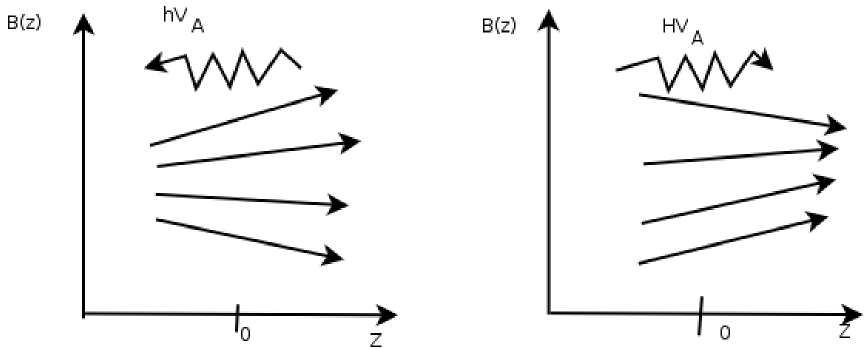

Figure 3: Conditions for 1st-order distributed Fermi acceleration in diverging (a) and converging (b) guide magnetic fields. In diverging magnetic fields a net negative ($h = -H_c < 0$) cross helicity state of Alfven waves (pronged curve) convects the average particle to regions of stronger field stength. In converging magnetic fields a net positive ($H_c > 0$) cross helicity state of Alfven waves also convects the average particle to regions of stronger field stength. In both cases the conservation of the pitch-angle averaged magnetic moment of the particle requires the increase of the particle momentum.