

# THREE DIMENSIONAL MODEL OF THE INTERPLANETARY MAGNETIC FIELD AND 27-DAY VARIATION OF THE GALACTIC COSMIC RAY INTENSITY

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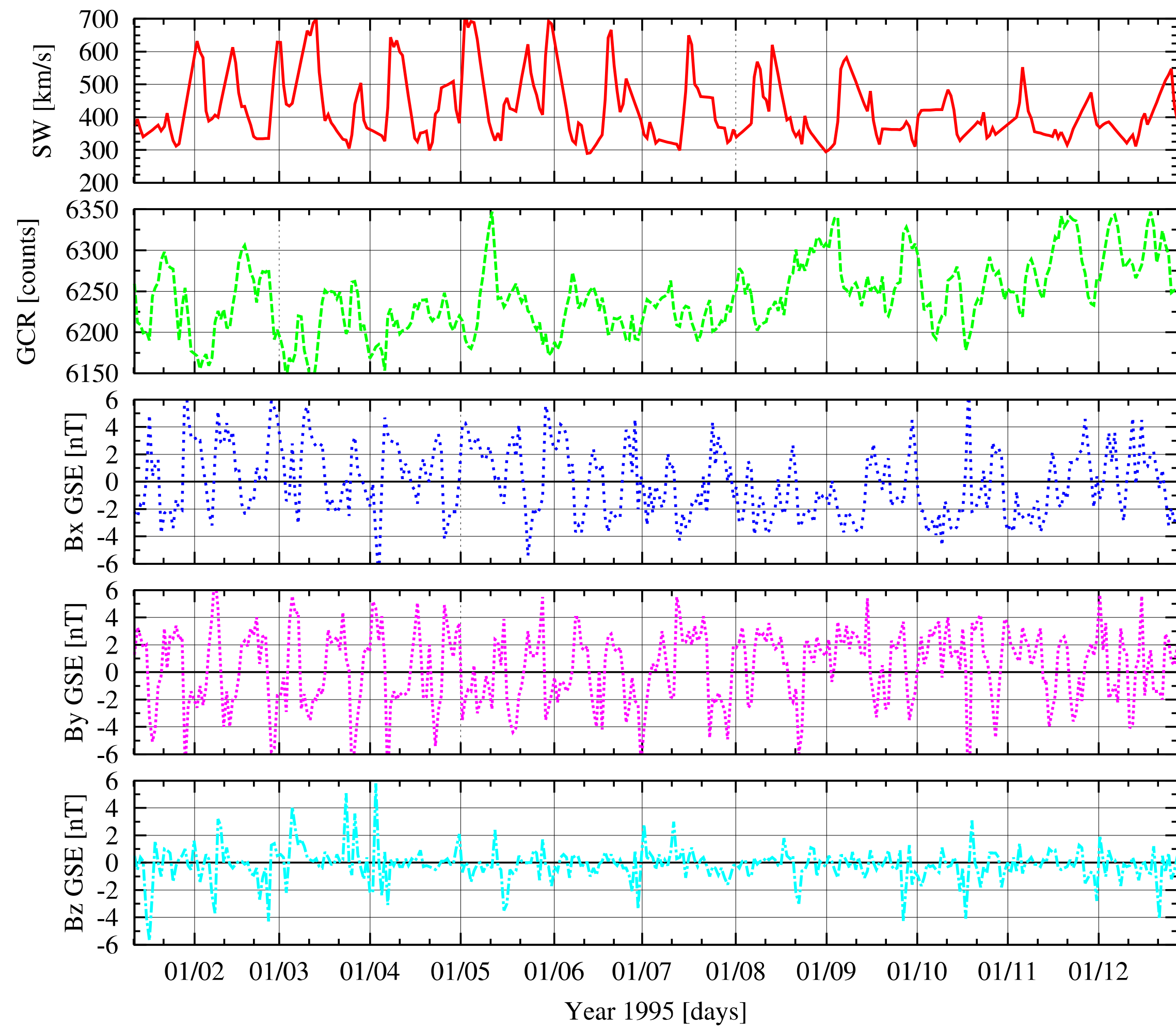
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## ABSTRACT

Numerical solutions of Maxwell's equations for three dimensional (3-D) heliosphere with the heliolongitudinal and heliolatitudinal dependent radial solar wind speed for minimum epoch of solar activity are presented. We show that an inclusion of the spatial distributions of  $B_r$  and  $B_\theta$  components of the interplanetary magnetic field (IMF) makes 3-D models of the 27-day variation of the galactic cosmic ray (GCR) intensity more realistic and compatible with experimental data. As a source of the 27-day variation of the GCR intensity is considered a heliolongitudinal asymmetry of the solar wind velocity. Problem of intersections of the magnetic field lines is considered in case of interactions of the fast and slow streams of solar wind plasma.

## 1. Experimental data

In this paper we analyze experimental data of the daily solar wind velocity, GCR intensity from the Kiel neutron monitor and radial  $B_x$ , azimuthal  $B_y$  and heliolatitudinal  $B_z$  components of the IMF in the minimum epoch of solar activity for the period of 1995.



**Figure 1.** Temporal changes of the daily solar wind velocity (SW) [OMNI], GCR intensity from the Kiel neutron monitor and radial  $B_x$ , azimuthal  $B_y$  and latitudinal  $B_z$  components of the IMF [OMNI] for the minimum epoch of solar activity in the period of 1995.

## 2. On the modeling of the 27-day variation of the GCR intensity

For modeling the 27-day variation of the GCR intensity we use stationary  $\frac{N}{r}$  Parker's transport equation (Parker, 1965):

$$i K_{ij} \frac{\partial N}{\partial x_j} - i V_i N \frac{\partial N}{\partial r} - \frac{1}{3} \frac{\partial}{\partial r} (N R) - i V_i = 0 \quad (1)$$

Where  $N$  and  $R$  are density and rigidity of cosmic ray particles, respectively;  $V_i$  solar wind velocity,  $K_{ij}$  is the anisotropic diffusion tensor of cosmic rays taken from (Alania, 2002).

In the proposed model we assume that the stationary 27-day variation of the GCR intensity is caused by the heliolongitudinal asymmetry of the solar wind speed corresponding to the in situ measurements (Fig. 2). We assume also that solar wind velocity changes versus heliolatitude as is presented by Ulysses data (McComas et al., 2002) (Fig. 3). Presented in Fig. 2 are the daily data of the solar wind speed (points) and dashed curve represents the approximation of the first and second harmonic waves (27 and 14-day variations) during the period of 1995.

An approximation of the changes of the daily solar wind speed (dashed line in Fig. 2) has a form

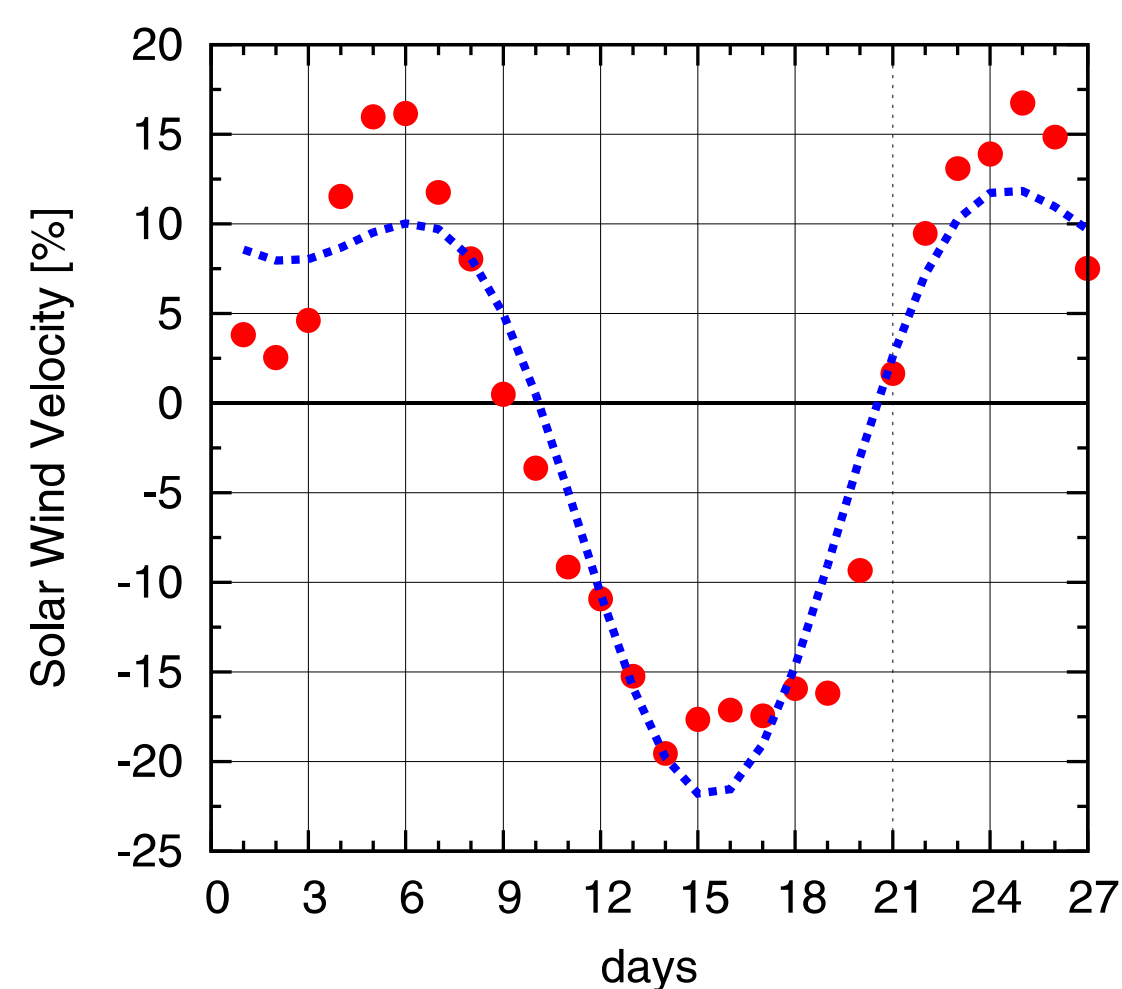
$$V_r = V_0 (1 + 0.15 \sin(\theta) + 1.18 \sin(2\theta) + 0.07 \sin(2\theta - 0.64)) \quad (2)$$

Where  $V_r(\theta) = V_0 (1 + 0.15 \sin(\theta) + 1.18 \sin(2\theta) + 0.07 \sin(2\theta - 0.64))$

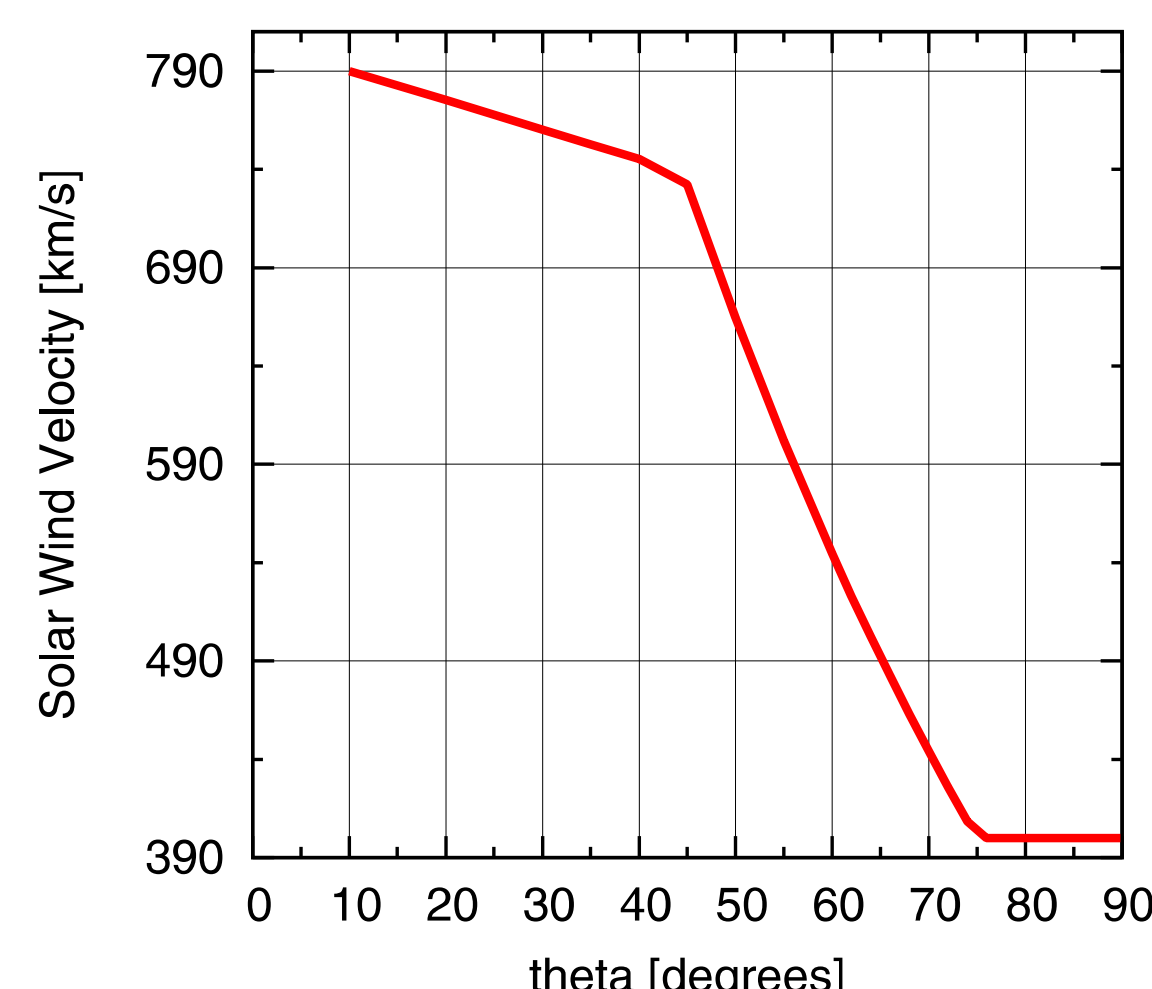
and  $V_0$  as is presented in the Fig. 3.

This expression (2) of the daily solar wind speed is implemented in Eq. (1).

We also take into account the heliolatitudinal dependence of the radial solar wind speed for minimum epoch of solar activity in 1995 according to Ulysses observations (Fig. 3).



**Figure 2.** Temporal changes of the daily solar wind velocity at the Earth orbit (points) by means of 13 Carrington rotations and dashed curve representing the sum of two harmonics (27 and 14 days) waves for the 1995 ( $V_r(\theta)$  in (2)).



**Figure 3.** Heliolatitude dependence of the radial solar wind speed for minimum epoch of solar activity in 1995 according to Ulysses observations ( $V_r(\theta)$  in (2)).

To solve the equation (1) there is necessary to implement the  $B_x$ ,  $B_y$  and  $B_z$  components of the IMF corresponding to the changeable solar wind velocity; for this purpose the Maxwell's equations should be solved for the solar wind velocity represented by formula (2). We consider Maxwell's equations:

$$\frac{\partial B}{\partial t} = -\text{grad} \phi \quad (3a)$$

$$\text{div} B = 0 \quad (3b)$$

where  $B$  is the IMF,  $V$  solar wind velocity, and  $t$ -time. The system of scalar equations for the components of the IMF and components of the solar wind speed corresponding to Eqs. (3a) and (3b) can be rewritten in the heliocentric spherical coordinate system, as:

$$\frac{\partial B_r}{\partial t} - [(V_\theta B_r - V_r B_\theta)] r \sin \theta - [(V_r B_r - V_\theta B_\theta)] r \quad (4a)$$

$$\frac{\partial B_\theta}{\partial t} - (V_r B_\theta - V_\theta B_r) - \frac{1}{r} [(V_r B_r - V_\theta B_\theta)] r \sin \theta \quad (4b)$$

$$\frac{\partial B}{\partial t} - [(V_r B_r - V_\theta B_\theta)] r - (V_r B_r - V_\theta B_\theta) \quad (4c)$$

$$\frac{1}{r} \frac{\partial}{\partial r} (r^2 B_r) - \frac{1}{\sin \theta} \frac{\partial}{\partial \theta} (\sin \theta B_\theta) - \frac{1}{r \sin \theta} \frac{\partial B}{\partial \theta} = 0 \quad (4d)$$

## 2.1. Numerical solution of Maxwell's equations

We assume that the changes of the solar wind velocity, the GCR intensity,  $B_x$ ,  $B_y$  and  $B_z$  components of the IMF are quasi stationary, i.e. the distribution of the GCR density is determined by the time independent parameters. Therefore, we accept that in Eqs. (4a)-(4c)  $\frac{\partial}{\partial t} = 0$ .

Also, we accept that average value of the heliolatitudinal  $V_\theta$  component of the solar wind velocity equals zero; then the system of Eqs. (4a)-(4d) can be reduced, as

$$\sin V_r \frac{\partial B_r}{\partial r} \sin B_\theta \frac{\partial B_\theta}{\partial \theta} \cos V_r B_r V_\theta \frac{\partial B}{\partial \theta} - V_r \frac{\partial B}{\partial r} - V_\theta \frac{\partial B}{\partial \theta} = 0 \quad (5a)$$

$$V_r \frac{\partial B}{\partial r} - V_\theta \frac{\partial B}{\partial \theta} - r \sin V_r \frac{\partial B_r}{\partial r} - r \sin B_\theta \frac{\partial B_\theta}{\partial \theta} - \frac{V_r}{r} \sin V_r B_\theta = 0 \quad (5b)$$

$$r B_r \frac{\partial V_r}{\partial r} - r V_\theta \frac{\partial B_\theta}{\partial \theta} - V_r B_r V_\theta B_\theta - r V_r \frac{\partial B_r}{\partial r} - r B_\theta \frac{\partial V_\theta}{\partial \theta} - r B_\theta \frac{\partial V_r}{\partial r} - V_r \frac{\partial B}{\partial r} - V_\theta \frac{\partial B}{\partial \theta} = 0 \quad (5c)$$

$$\frac{B_r}{r} \frac{\partial}{\partial r} (r^2 B_r) - \frac{1}{\sin B_\theta} \frac{\partial}{\partial \theta} (\sin B_\theta B_\theta) - \frac{1}{r \sin B_\theta} \frac{\partial B}{\partial \theta} = 0 \quad (5d)$$

The latitudinal component of the IMF is very feeble for the period to be analyzed, so we can assume that it equals zero, so further we consider 2D model of the interplanetary magnetic field. This assumption straightforwardly leads (from Eq. (4a)) to the relationship between  $B_r$  and  $B_\theta$ , as  $B_\theta = B_r \frac{V_r}{V_\theta}$ . Then Eq. (5d) with respect to the radial component has a form:

$$A_1 \frac{B_r}{r} - A_2 \frac{B_r}{r} - A_3 B_r = 0 \quad (6)$$

We take into account, as well:

$$V_\theta = 0, \quad V_r = r \sin \theta \quad (7)$$

Where  $V_\theta$  is the negative corotational speed and  $\omega$  is the angular velocity of the Sun.

Taking into consideration the expressions (2) and (7) the coefficients in Eq. (6) are:  $A_1 = 1, A_2 = \frac{V_r}{r}, A_3 = \frac{2}{r} \frac{V_r}{V_\theta}$ . We solve Eq. (6) by numerical method. Equation (6) was reduced to the algebraic system of equations using a difference scheme method (e.g., Kincaid and Cheney, 2006), as

$$A_1 \frac{B_r[i-1, j, k]}{r} - A_2 \frac{B_r[i, j, k]}{r} - A_3 B_r[i, j, k] = 0 \quad (8)$$

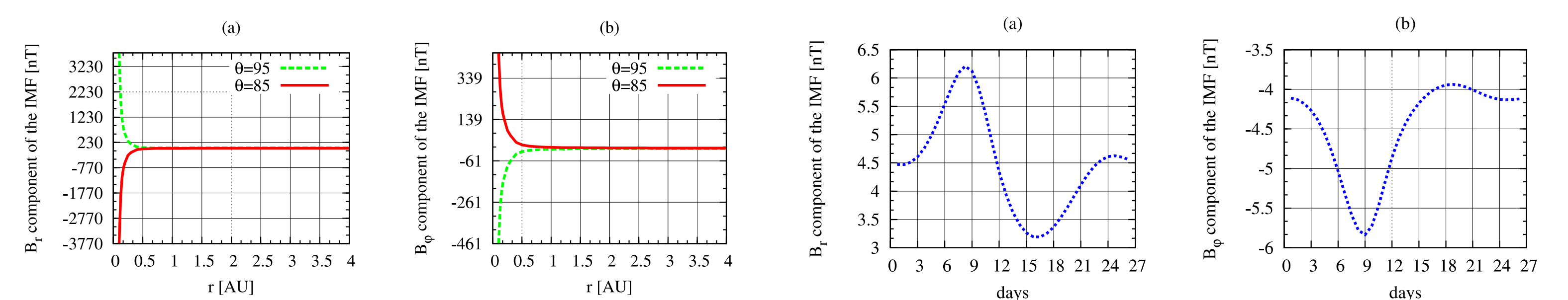
where,  $i=1, 2, \dots, I; j=1, 2, \dots, J; k=1, 2, \dots, K$  are steps in radial distance, vs. heliolatitude and heliolongitude, respectively. Eq. (8) was solved by the iteration method with the boundary condition near the Sun  $B_r[1, j, k] = const$ ; in considered case  $r_1 = 0.1 \text{ AU}$ ,

$$B_r[1, j, k] = \begin{cases} 3770 \text{ nT} & \text{for } \theta^0 = 90^\circ \\ 3770 \text{ nT} & \text{for } \theta^0 = 180^\circ \end{cases}$$

for the positive polarity period ( $A > 0$ ).

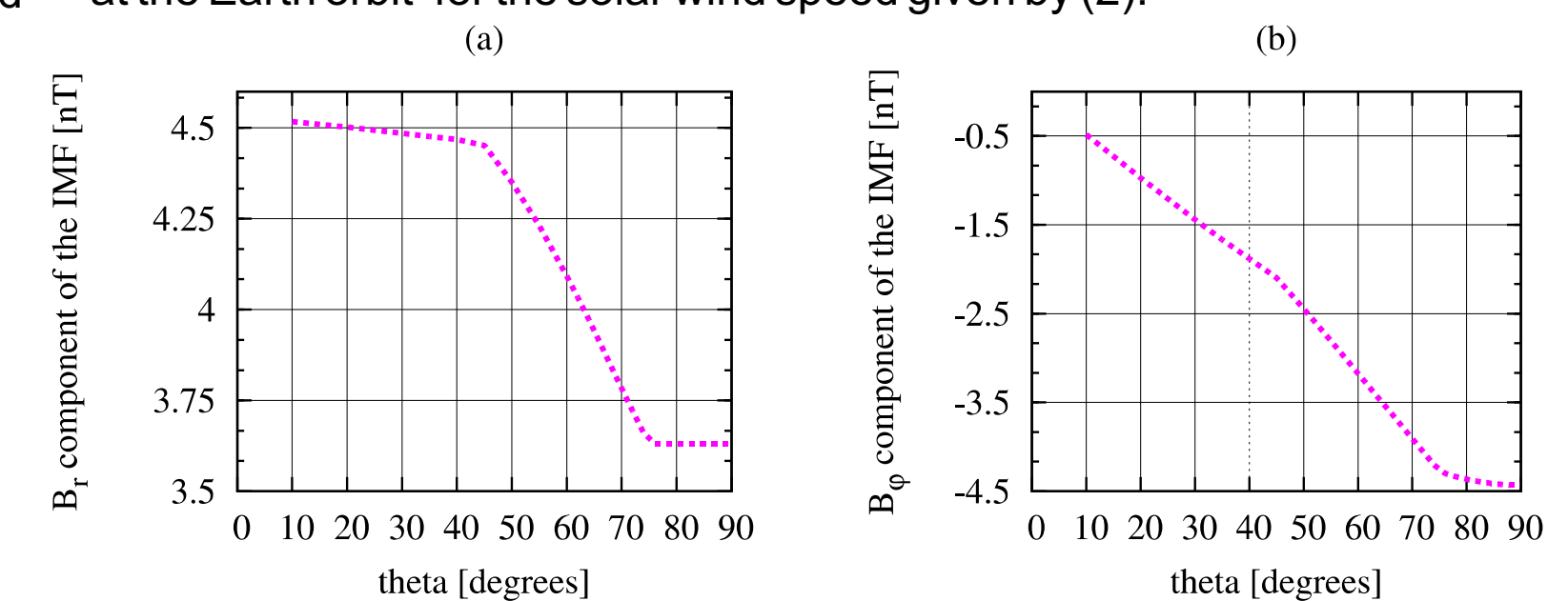
The choice of these boundary conditions was stipulated by requiring agreement of the solutions of Eq. (8) with the in situ measurements of the  $B_r$  and  $B_\theta$  components of the IMF at the Earth orbit.

Presented in Figs. 4-6 are results of the solution of Eq. (8) for the  $B_r$  and  $B_\theta$  components of the IMF.



**Figure 4a.** Radial changes of the  $B_r$  and  $B_\theta$  components of the IMF for different heliolatitudes near the solar equatorial plane for the solar wind speed given by (2).

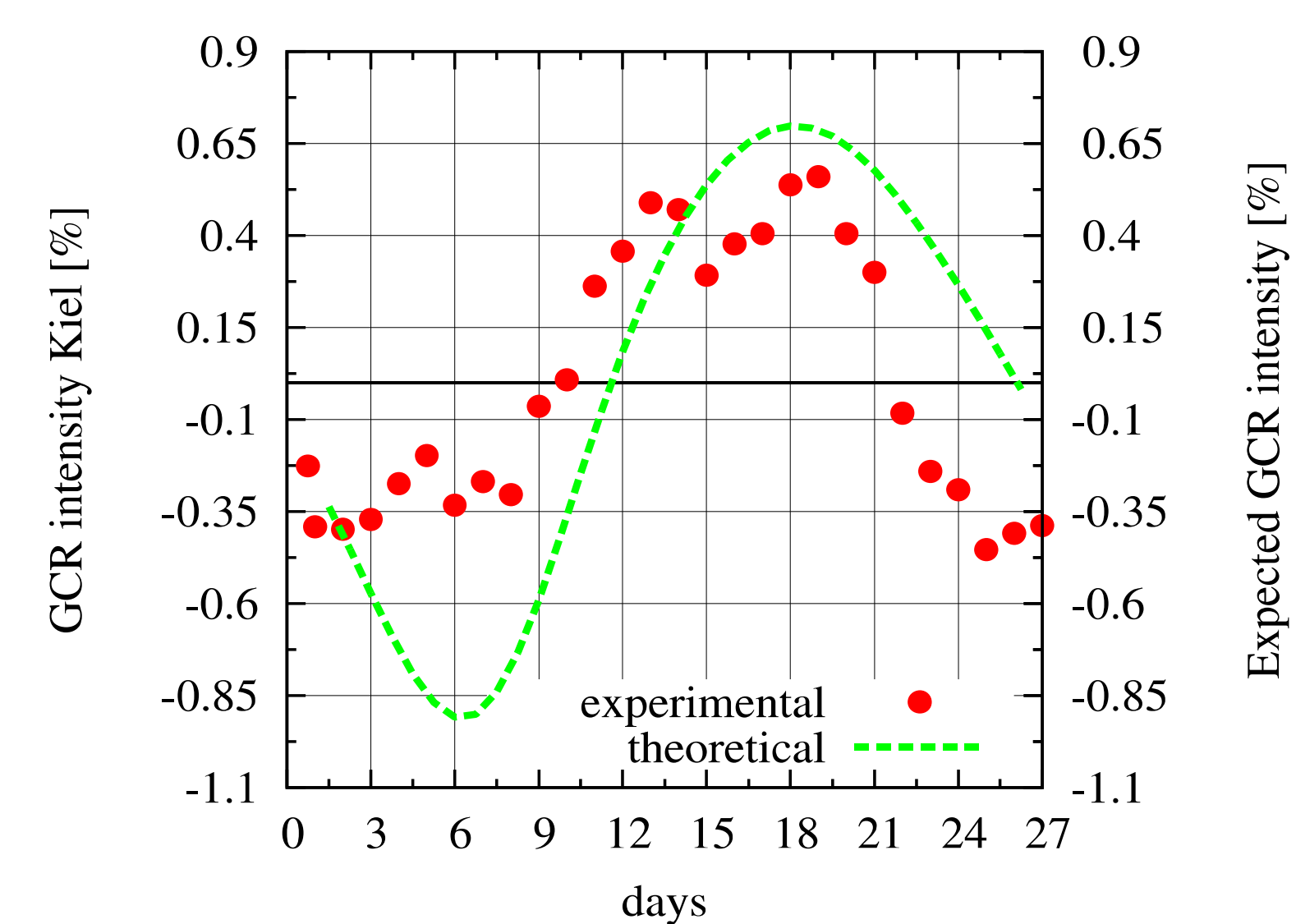
**Figure 5a.** Azimuthal changes of the  $B_r$  and  $B_\theta$  components of the IMF at the Earth orbit for the solar wind speed given by (2).



**Figure 6a.** Heliolatitude changes of the  $B_r$  and  $B_\theta$  component of the IMF at 1 AU for the solar wind speed given by (2).

## 2.2. Numerical solution of the model of the 27-day variation of the GCR intensity

In Parker's transport equation (1) we included the components  $B_r$  and  $B_\theta$  and the magnitude  $B$  of the IMF obtained from the numerical solution of Eq. (8) with a variable solar wind speed. Implementation of the heliospheric magnetic field obtained from the numerical solution of Eq. (8) in Parker's transport equation is done through the spiral angle in anisotropic diffusion tensor of GCR particles and ratios of  $K_{\parallel}/K_{\perp}$  and  $K_{\theta}/K_{\parallel}$ . The kinematical model of the IMF with variable solar wind speed has some limitations, especially it would be applied until some radius, while at large radii the faster wind would overtake the previously emitted slower one. To exclude an intersection of the IMF lines the heliolongitudinal asymmetry of the solar wind speed takes place only up to the distance of 8 AU and then  $V=400 \text{ km/s}$  throughout the heliosphere. In connection with this behind 8 AU in the theoretical model standard Parker's field is used. Equation (1) was solved numerically as in our papers published elsewhere (e.g., Alania, 2002; Iskra et al., 2004; Modzelewska et al., 2006; Wawrzynczak and Alania, 2008). Changes of the relative density obtained as a solution of the transport Eq. (1) for the model of the 27-day variation of the GCR intensity corresponding to the analyzed period are presented in Fig. 7 (dashed line); in this figure are also presented (points) changes of the GCR intensity obtained by Kiel neutron monitor experimental data for the period of 1995 (Fig. 1), as well. Fig. 7 shows that results of theoretical modeling (dashed line) and the experimental data (points) are in good agreement. We underline that the presented model of the 27-day variation of the GCR intensity composed for the variable solar wind speed (2) and the components of the IMF obtained as the solution of Eq. (8) is compatible with the neutron monitors experimental data.



**Figure 7.** Heliolongitudinal changes of the expected GCR intensity for effective rigidity 10-15 GV at the Earth orbit during solar rotation period (dashed line) and temporal changes of the superimposed GCR intensity by Kiel neutron monitor during 27 day for the period of 1995 (points) for the solar wind velocity assumed as in expression (2).

## CONCLUSIONS

1. The quasi steady 27-day variations of the solar wind velocity, GCR intensity,  $B_r$  and  $B_\theta$  components of the IMF have been analyzed for the period of 1995.

2. The Maxwell equations are solved with a solar wind speed varying in heliolongitude and heliolatitude (based on Ulysses data) in accordance with in situ measurements in case to derive the longitudinal dependence of the  $B_r$  and  $B_\theta$  components of the IMF.

3. A three-dimensional model is proposed for the 27-day variation of GCR intensity in response to a realistic variation of the solar wind velocity. The model incorporates the  $B_r$  and  $B_\theta$  components of the IMF derived from the Maxwell equations.

4. The proposed model of the 27-day variation of the GCR intensity is in good agreement with the experimental data of Kiel neutron monitor.

## ACKNOWLEDGMENTS

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