The heliolongitudinal dependence of solar wind velocity and the second harmonic of the 27-day variation of cosmic rays

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ABSTRACT

We study the period of 10 June - 29 August 1994 (2197-2199 Carrington rotations) when second harmonic of the 27-day wave in the solar wind speed and galactic cosmic ray intensity is well manifested. We develop a three dimensional (3-D) model of the second harmonic of the 27-day variation of galactic cosmic ray (GCR) intensity with the heliolongitudinal dependence of the solar wind velocity. Maxwell equations are numerically solved to derive a divergence-free interplanetary magnetic field with variable solar wind speed reproducing in situ observations. The proposed model for the second harmonic of the 27-day variation of the GCR intensity is based on the Parker transport equation using the changeable solar wind velocity and the corresponding magnetic field. The predictable profile of the second harmonic of the 27-day variation of cosmic ray intensity is compatible with neutron monitors experimental data. We show that the time profile of the second harmonic of the 27-day variation of cosmic ray intensity is inversely correlated with the modulation parameter ζ being proportional to the product of the solar wind velocity V and the interplanetary magnetic field strength B (ζ ~ VB).

1. INTRODUCTION

Theoretical study of different classes of the galactic cosmic rays (GCR) intensity variations generally is implied by Parker's transport (Parker, 1965) equation with the constant solar wind velocity, and for the interplanetary magnetic field (IMF) B satisfying, equation $\nabla \cdot B = 0$. To properly model the 27-day variation of cosmic rays, the spatial dependence of the solar wind velocity V, and the IMF must be taken into account. However, it is rather complicated problem, because the validity of the Maxwell's equation $\nabla \cdot B = 0$ should be kept for the spatially dependent solar wind speed (see Alania et al., 2010 and references therein). Many of the papers (Gil and Alania, 2001; Alania et al., 2005; Gil et al., 2005; Alania et al., 2008a, 2008b; Kota and Jokipii, 2001; Burger and Hitge, 2004; Burger et al., 2008) aimed to explain the distinction between amplitudes of the 27-day variation vs. magnetic polarity noticed by Richardson et al. (1999), by predictions of drift theory and the role of recurrent changes of the solar wind speed, which is a crucial (Alania et al., 2008a; Gil et al., 2008), was not considered. In this paper we perform model calculations for the second harmonic of the 27-day variation of the GCR intensity using the variable solar wind and the corresponding magnetic field derived from the solution of the Maxwell equations. To properly construct the model of the second harmonic of the 27-day variation of the GCR intensity the spatial dependence of the solar wind velocity and the interplanetary magnetic field must be taken into account. We consider Maxwell's equations (e. g. Parker, 1963; Jackson, 1998):

$$\frac{\partial B}{\partial t} = \nabla \times (V \times B) \qquad (1a)$$

$$\nabla \cdot B = 0 \qquad (1b)$$

The system of scalar equations for the components B_r, B_θ, B_φ of the IMF and components V_r, V_θ, V_φ of the solar wind speed corresponding to Eq. (1a) and condition $\nabla \cdot B = 0$ can be rewritten in corotating frame (attached to the rotating Sun) in the heliocentric spherical r, θ, φ coordinate system, as:

$$\frac{\partial B_r}{\partial t} = \frac{1}{r^2 \sin\theta}\left\{\frac{\partial}{\partial \theta}[(V_r B_\theta - V_\theta B_r) r \sin\theta] - \frac{\partial}{\partial \varphi}[(V_\varphi B_r - V_r B_\varphi) r]\right\} \qquad (2a)$$

$$\frac{\partial B_\theta}{\partial t} = \frac{1}{r \sin\theta}\left\{\frac{\partial}{\partial \varphi}(V_\theta B_\varphi - V_\varphi B_\theta) - \frac{\partial}{\partial r}[(V_r B_\theta - V_\theta B_r) r \sin\theta]\right\} \qquad (2b)$$

$$\frac{\partial B_\varphi}{\partial t} = \frac{1}{r}\left\{\frac{\partial}{\partial r}[(V_\varphi B_r - V_r B_\varphi) r] - \frac{\partial}{\partial \theta}(V_\theta B_\varphi - V_\varphi B_\theta)\right\} \qquad (2c)$$

$$\frac{1}{r}\frac{\partial}{\partial r}(r^2 B_r) + \frac{1}{\sin\theta}\frac{\partial}{\partial \theta}(\sin\theta B_\theta) + \frac{1}{\sin\theta}\frac{\partial B_\varphi}{\partial \varphi} = 0 \qquad (2d)$$

Our aim in this paper is to compose a model of the second harmonic of the 27-day variation of the GCR intensity for the solar wind speed depending on heliolongitude reproducing in situ measurements. We consider the time interval corresponding to special conditions in the interplanetary space, 10 June -29 August 1994 with apparent second harmonic (14 day) of the 27-day variation in the solar wind speed and in the GCR intensity.

2. DATA ANALASIS

We analyze experimental data of the daily solar wind velocity, GCR intensity from the Moscow neutron monitor and radial Bx, azimuthal By and heliolatitudinal Bz components of the IMF, Wolf number and solar radio flux for the time interval 10 June-29 August 1994 (2197-2199 Carrington Rotations) when the second (14 days) harmonic of the 27-day wave in the solar wind velocity and galactic cosmic ray intensity is well recognized. Presented in Fig. 1 are the temporal changes of the solar wind speed [SPIDR http://spidr.ngdc.noaa.gov/spidr/], GCR intensity from the Moscow neutron monitor [http://helios.izmiran.troitsk.ru/cosray/main.htm] and Bx, By and Bz components of the IMF [OMNI http://omniweb.gsfc.nasa.gov/index.html], Wolf number [http://www.ngdc.noaa.gov/stp/SOLAR/ftpsunspotnumber.html] and solar radio flux [http://www.ngdc.noaa.gov/stp/SOLAR/ftpsolarradio.html] for the time interval 10 June - 29 August 1994 corresponding to 2197-2199 Carrington rotations (CR). In Fig. 1 "0" value of the IMF components designates missing data of the IMF.

As it was shown (Gil and Alania, 2001; Modzelewska et al., 2006; Alania et al., 2008a), the heliolongitudinal asymmetry of the solar wind speed is one of the crucial parameters in creation of the 27-day variation of the GCR intensity in the minimum and near minimum epochs of solar activity. In connection with this, we estimate a contributions of each of the three harmonics (27, 14 and 9 days) in the daily changes of the solar wind velocity and GCR intensity in the time interval to be analyzed (Fig. 2ab). In this purpose we determine the best fit sine wave to each group of data using frequency filter method (e.g., Otnes and Enochson, 1972). This technique decomposes a time series into frequency components. We use band pass filter selecting only the components with a period (frequency) within these bounds. A band-pass filter rejects high and low frequencies, passing only signal around some intermediate frequency. The frequency-domain behavior of a filter is described mathematically in terms of its transfer function or network function. This is the ratio of the Laplace transforms of its output and input signals. We investigate periodicity bound within 24-32 days (27-28 days in the middle) for the I harmonic, 11-17 days (14 days in the middle) for II harmonic and 6-12 days (9 days in the middle) for III harmonic of the 27-day wave. This procedure have been performed for solar wind speed and GCR intensity from the Moscow neutron monitor. Results are presented in Fig. 2ab).

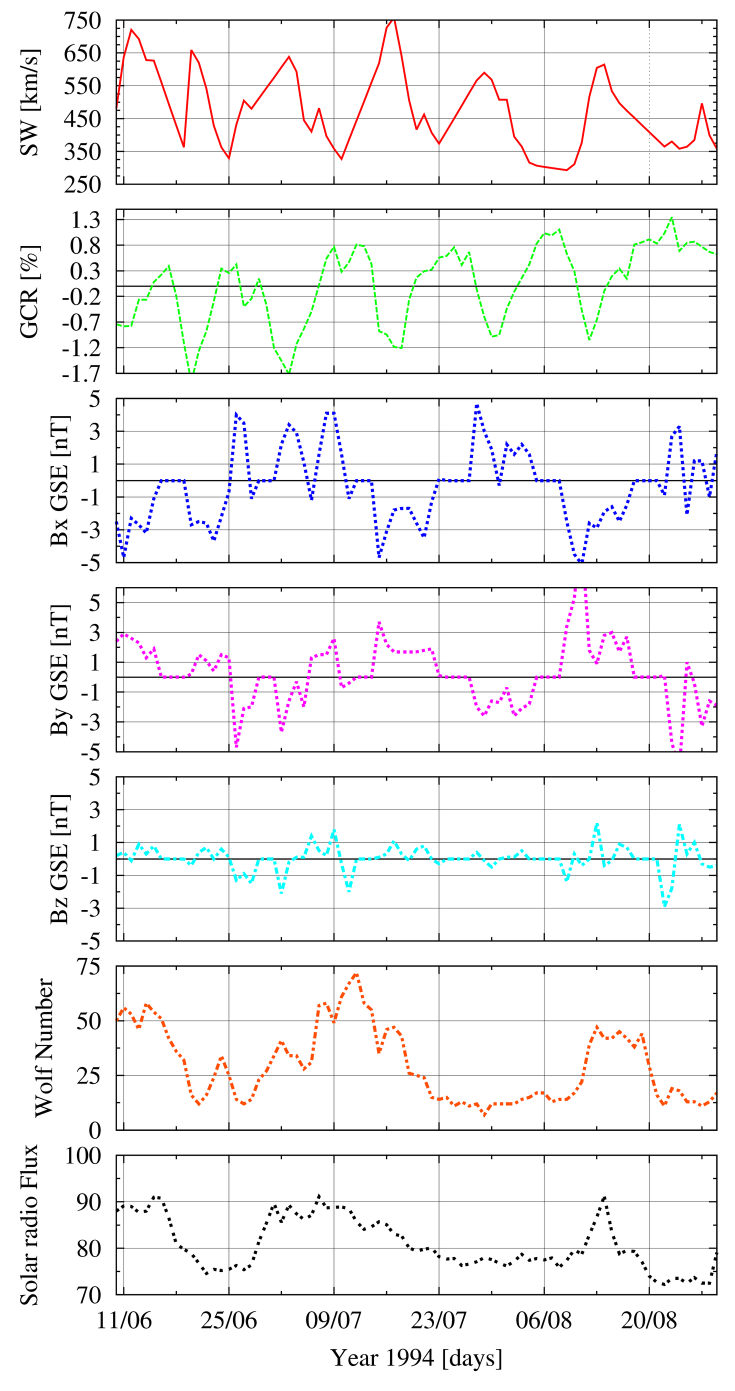

Figure 1 . Temporal changes of the daily solar wind velocity (SW) [SPIDR], GCR intensity from the Moscow neutron monitor and radial Bx, azimuthal By and latitudinal Bz components of the IMF [OMNI], Wolf number and solar radio flux for the period of 10 June - 29 August 1994; 2197-2199 CR.

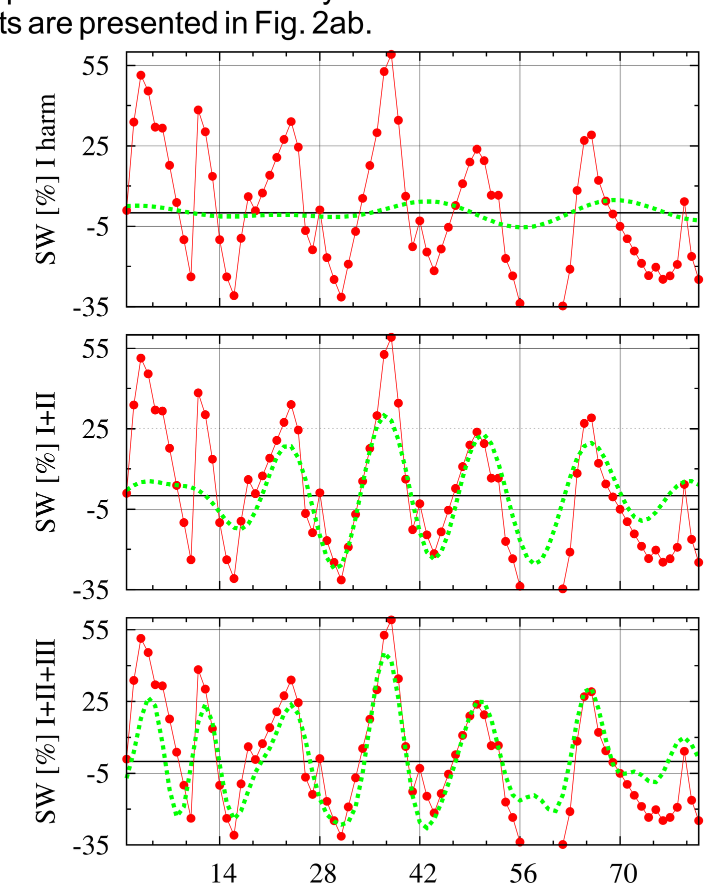

Figure 2a Temporal changes of daily solar wind speed (solid lines) and the first harmonic (27 days) wave (top panel), sum of I and II harmonic (14 days) waves (middle panel), and sum of I, II and III harmonic (9 days) waves (bottom panel) for the period of 10 June -29 August 1994

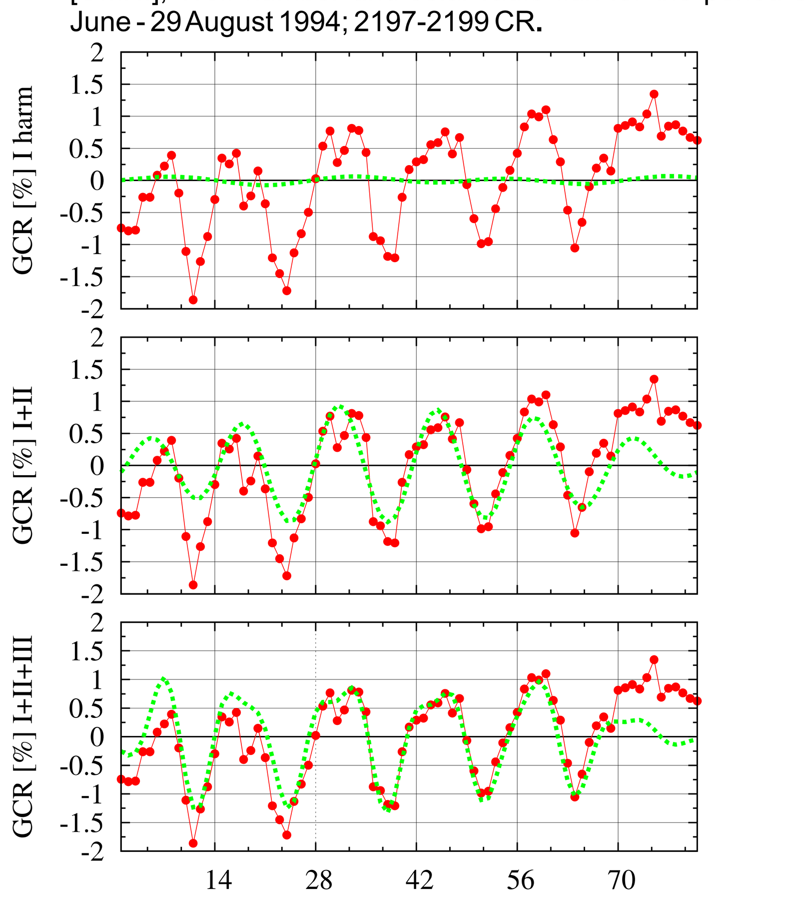

Figure 2b Temporal changes of daily GCR intensity (solid lines) and the first harmonic (27 days) wave (top panel), sum of I and II harmonic (14 days) waves (middle panel), and sum of I, II and III harmonic (9 days) waves (bottom panel) for the period of 10 June -29 August 1994

10 June-29 August 1994	Solar wind speed	GCR intensity
I harm wave	0.36±0.10	0.24±0.11
I+II harm wave	0.74±0.08	0.72±0.08
I+II+III harm wave	0.81±0.07	0.80±0.07

Table Correlation coefficients between the observed data and first harmonic wave, observed data and the sum of I and II harmonic waves, observed data and the sum of I, II and III harmonic waves (Fig. 2ab)

3.1. NUMERICAL SOLUTION OF MAXWELL'S EQUATIONS

We assume that the changes of the solar wind velocity, the GCR intensity, Bx, By and Bz components of the IMF are quasi stationary , i.e. the distribution of the GCR density is determined by the time independent parameters. Therefore, we accept that in Eqs. (2a)-(2c) $\frac{\partial B_r}{\partial t} = 0, \frac{\partial B_\theta}{\partial t} = 0, \frac{\partial B_\varphi}{\partial t} = 0$

Also, we accept that average value of the heliolatitudinal component of the solar wind velocity V_θ equals zero; then the system of Eqs. (2a)-(2d) can be reduced, as

$$\sin\theta\, V_r \frac{\partial B_\theta}{\partial \theta} - \sin\theta\, B_\theta \frac{\partial V_r}{\partial \theta} - \cos\theta\, V_r B_\theta + V_\varphi \frac{\partial B_r}{\partial \varphi} - B_r \frac{\partial V_\varphi}{\partial \varphi} - V_r \frac{\partial B_\varphi}{\partial \varphi} + B_\varphi \frac{\partial V_r}{\partial \varphi} = 0 \qquad (3a)$$

$$V_\varphi \frac{\partial B_\theta}{\partial \varphi} - B_\theta \frac{\partial V_\varphi}{\partial \varphi} + r\sin\theta\, V_r \frac{\partial B_\theta}{\partial r} - r\sin\theta\, B_\theta \frac{\partial V_r}{\partial r} + \sin\theta\, V_r B_\theta = 0 \qquad (3b)$$

$$rB_r \frac{\partial V_\varphi}{\partial r} - rV_\varphi \frac{\partial B_r}{\partial r} + V_\varphi B_r - V_r B_\varphi - rV_r \frac{\partial B_\varphi}{\partial r} + rB_\varphi \frac{\partial V_r}{\partial r} + B_\theta \frac{\partial V_\varphi}{\partial \theta} - V_\varphi \frac{\partial B_\theta}{\partial \theta} = 0 \qquad (3c)$$

$$\frac{\partial B_r}{\partial r} + \frac{2}{r} B_r + \frac{ctg\theta}{r} B_\theta + \frac{1}{r}\frac{\partial B_\theta}{\partial \theta} + \frac{1}{r\sin\theta}\frac{\partial B_\varphi}{\partial \varphi} = 0 \qquad (3d)$$

The latitudinal component of the IMF is very feeble for the period to be analyzed, so we can assume that it equals zero , so further we consider 2D model of the interplanetary magnetic field. This assumption straightforwardly leads (from Eq. (2a)) to the relationship between B_r and B_φ , as, $B_\varphi = B_r \frac{V_\varphi}{V_r}$. Then Eq. (3d) with respect to the radial component has a form:

$$A_1 \frac{\partial B_r}{\partial r} + A_2 \frac{\partial B_r}{\partial \varphi} + A_3 B_r = 0 \qquad (4)$$

We include in Eq. (4) radial solar wind speed as the sum of three harmonic waves (dashed curve in Fig.3) approximated by the formula:

$$V_r = V_0(1 + \alpha_1 \sin(\varphi + \varphi_1) + \alpha_2 \sin(2(\varphi + \varphi_2)) + \alpha_3 \sin(3(\varphi + \varphi_3))) \qquad (5)$$

$\alpha_1 = 0.05$, $\alpha_2 = 0.13$, $\alpha_3 = 0.08$, $\varphi_1 = 1.13$, $\varphi_2 = 0.1$, $\varphi_3 = 0.2$;

We take into account, as well:

$V_\theta = 0$, $V_\varphi = -\Omega r \sin\theta$

Where V_φ is the corotational speed and Ω is the angular velocity of the Sun.

The coefficients in Eq. (4) are: $A_1 = 1$, $A_2 = \frac{\Omega}{V_r}$, $A_3 = \frac{2}{r} - \frac{\Omega}{V_r^2}\frac{\partial V_r}{\partial \varphi}$

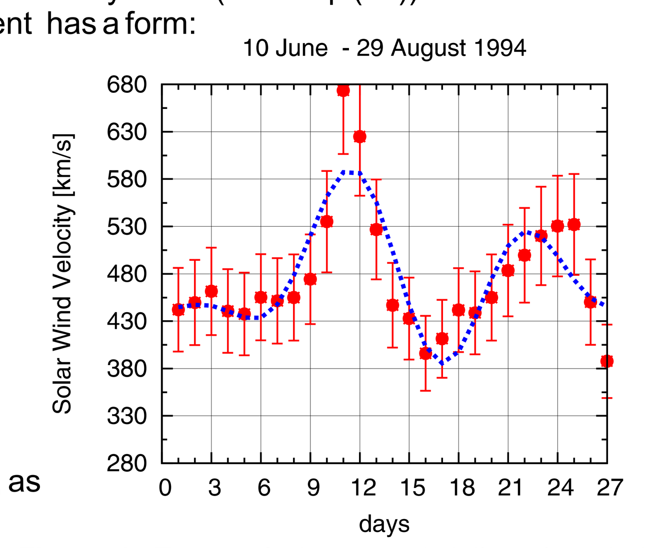

Figure 3. Temporal changes of daily solar wind speed (points) obtained by superposition of three Carrington rotations (2197-2199) corresponding to the period of 10 June - 29 August 1994, and the sum of three harmonic waves (dashed curve) of the solar wind speed.

We solve Eq. (4) by numerical method. Equation (4) was reduced to the algebraic system of equations using a difference scheme method (e.g., Kincaid and Cheney, 2006), as

$$A_1 \frac{B_r[i+1,j,k] - B_r[i,j,k]}{\Delta r} + A_2 \frac{B_r[i,j,k+1] - B_r[i,j,k]}{\Delta \varphi} + A_3 B_r[i,j,k] = 0 \qquad (6)$$

where, i=1,2,…, I; j=1,2,…, J; k=1,2,…, K are steps in radial distance, vs. heliolatitude and heliolongitude, respectively. Eq. (6) was solved by the iteration method with the boundary condition near the Sun.

In considered case r_1 =0.5 AU, $B_r[1,j,k] = 18\,nT$ for $0^0 \le \theta \le 90^0$; $-18\,nT$ for $90^0 < \theta \le 180^0$

for the positive polarity period (A>0).

The choice of these boundary conditions was stipulated by requiring agreement of the solutions of Eq. (6) with the in situ measurements of the components of the IMF at the Earth orbit.

Presented in Figs. 4-5 are results of the solution of Eq. (6) for the B_r and B_φ components of the IMF .

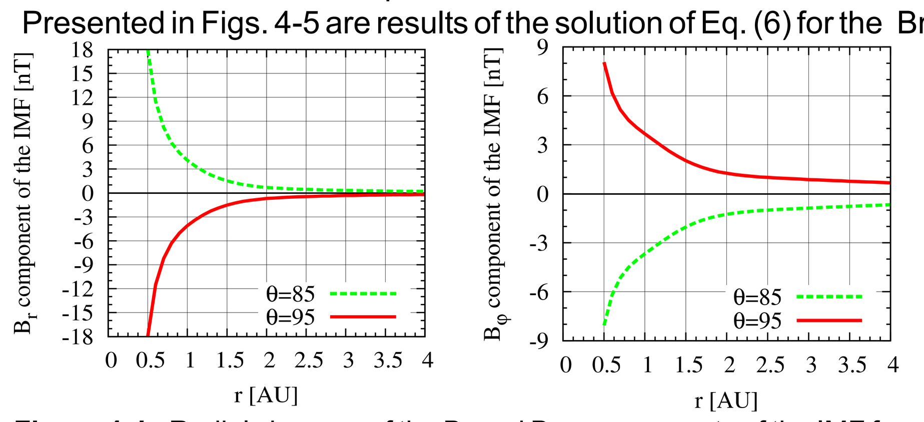

Figure 4ab. Radial changes of the Br and B_φ components of the IMF for different heliolatitudes near the solar equatorial plane for the solar wind speed given by (5).

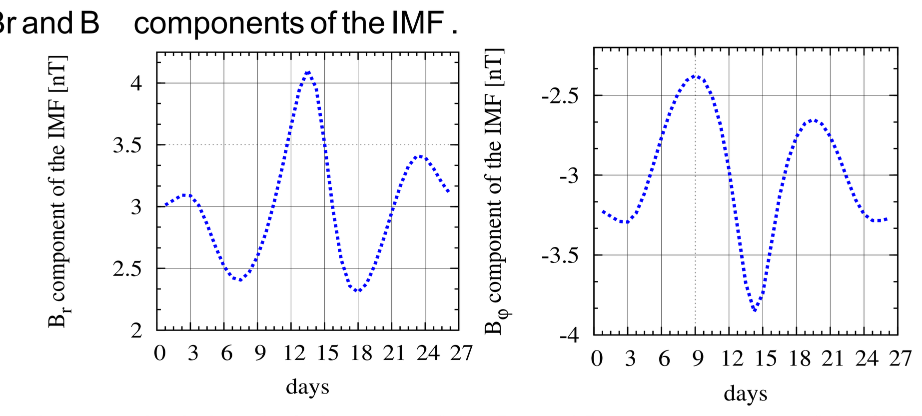

Figure 5ab. Azimuthal changes of the Br and B_φ components of the IMF at the Earth orbit for the solar wind speed given by (5).

3.2. ON THE MODELING OF THE SECOND HARMONIC OF THE 27-DAY VARIATION OF COSMIC RAYS

For modeling the 27-day variation of the GCR intensity we use stationary $\frac{\partial f}{\partial t} = 0$ Parker's transport equation (Parker, 1965):

$$\nabla \cdot (K_S \cdot \nabla f) - (v_d + V) \cdot \nabla f + \frac{1}{3}(\nabla \cdot V)\frac{\partial f}{\partial \ln R} = 0 \qquad (7)$$

Where f and R are omindirectional distribution function and rigidity of cosmic ray particles, respectively; V solar wind velocity, K is the anisotropic diffusion tensor of cosmic rays taken from (Alania, 2002).

The parallel diffusion coefficient changes versus the spatial spherical coordinates and rigidity R of GCR particles as,

$\kappa_\parallel = \kappa_0\, \kappa(r)\, \kappa(R)$, $\kappa(r) = 1 + \alpha_0 r$, α_0 =0.5, $\kappa(R) = (R/1GV)^{0.8}$, $\kappa_\parallel \approx 10^{23} \frac{cm^2}{s}$

In the model we assume that the stationary second harmonic of the 27-day variation of the GCR intensity is caused by the heliolongitudinal asymmetry of the solar wind speed. In Eq. (7), we included the changes of the solar wind speed (5) for the time interval to be analyzed. In Parker's transport equation we included also components and the magnitude of the IMF obtained from the numerical solution of Eq. (6) with a variable solar wind speed given by (5). Equation (7) was solved numerically as in our papers published elsewhere (e.g., Alania, 2002; Iskra et al., 2004; Modzelewska et al., 2006; Wawrzynczak and Alania, 2008; Alania et al., 2010). Results of the solution are presented in Fig. 6.

Generally it is worth to underline, that when diffusion coefficient K of the GCR particles depends on the regular IMF as K $\propto$ 1/B, there arises a modulation parameter ζ being proportional to the product of the solar wind velocity V and the strength B of the regular IMF, ζ ~ VB. In Fig. 7 are presented changes of the parameter ζ ~ VB (solid line) at the Earth orbit for the solar wind velocity V given by the expression (5) and magnitude B of the IMF obtained as the numerical solution of Eq. (6), and the expected second harmonic of the 27-day wave of the GCR intensity obtained as a solution of Parker's transport Eq. (7) for rigidity 10GV (dashed line). It is obvious that between these two curves is noteworthy negative correlation, confirming a significant influence of the modulation parameter ζ ~VB on the second harmonic of the 27-day variation of the GCR intensity; correlation coefficient equals -0.81.

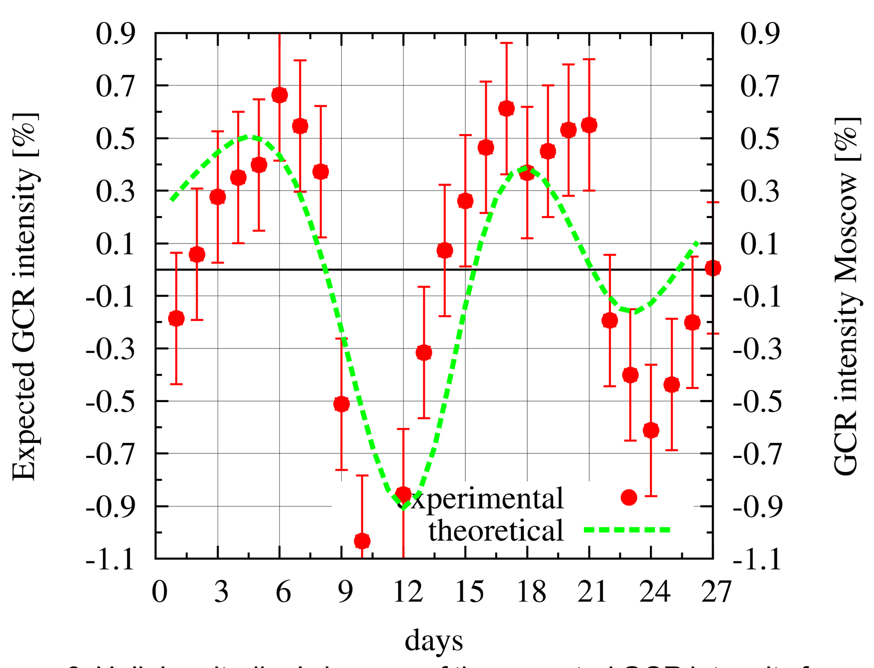

Figure 6. Heliolongitudinal changes of the expected GCR intensity for rigidity 10GV at the Earth orbit during solar rotation period (dashed line) and temporal changes of superimposed by three Carrington rotations GCR intensity from the Moscow neutron monitor for the period of 10 June - 29 August 1994 (points).

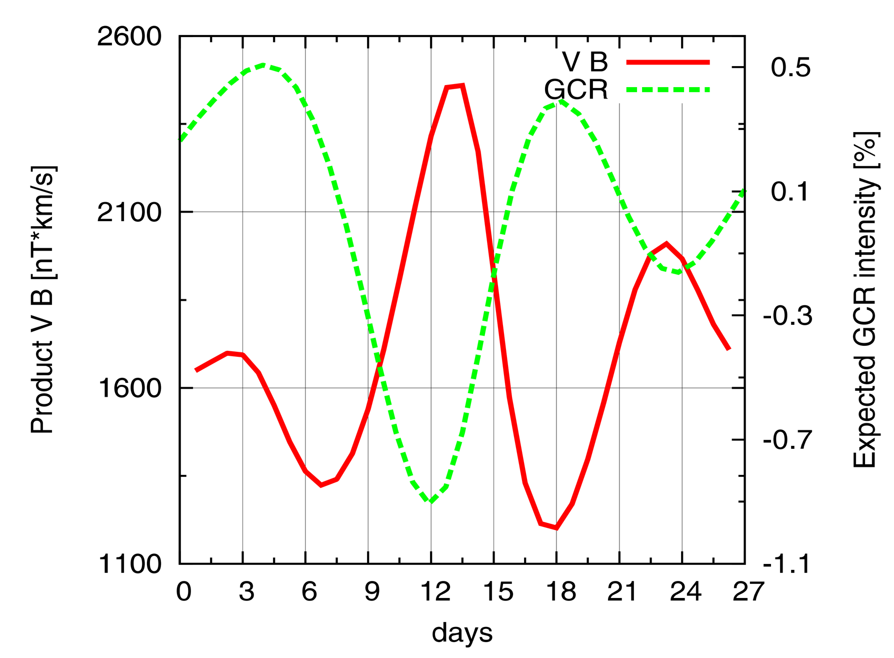

Figure 7. Heliolongitudinal changes of the parameter ζ ~ VB (solid line) and the expected GCR intensity for rigidity 10GV at the Earth orbit (dashed line) during solar rotation period.

CONCLUSIONS

1. The quasi steady second harmonic (14-days) of the 27-day variations of the solar wind velocity and GCR intensity have been analyzed for the period of 10 June-29 August 1994.

2. The Maxwell equations are solved with a solar wind speed varying in heliolongitude in accordance with in situ measurements in case to derive the longitudinal dependence of the Br and B_φ components of the IMF.

3. A three-dimensional model is proposed for the second harmonic of the 27-day variation of GCR intensity in response to a realistic variation of the solar wind velocity. The model incorporates the Br and B_φ components of the IMF derived from the Maxwell equations.

4. The proposed model of the second harmonic of the 27-day variation of the GCR intensity is in good agreement with the experimental data of Moscow neutron monitor.

5. On the basis of the convection-diffusion modulation model the expected second harmonic of the 27-day variation of the GCR intensity is inversely correlated with the modulation parameter ζ being proportional to the product of the solar wind velocity V and the strength of the interplanetary magnetic field B (ζ ~ VB). High anticorrelation (correlation coefficient equals -0.81) between them shows that the second harmonic of the 27-day variation of the GCR intensity takes place mainly due to this modulation effect in the minimum and near minimum epochs of solar activity.

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